Pareto Efficiency and the Competitive Market Economy.

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A great deal of this handout was originally written for undergraduates (in fact I require them to read and understand the first half). Thus the writing is not too technical. But that's fine, because it's the ideas—not the math—that is difficult here. In general welfare is an important issue to me. I feel that people who poorly understand what welfare is tend to use economics in a normative manner too much. Not that I understand what welfare is, I just know it is more than Pareto Efficiency, and want you to understand that too.

Pareto Efficiency is what too many economists mean when they say welfare. If something is Pareto Efficient it is "good," if it is not it is "bad." Now don't get me wrong. Knowing whether something is Pareto Efficient is very important, and feasible to do. I just want to make sure you understand *exactly* what it is and what its limitations are, and don't fall into sloppy usage.

Definition 1 *An allocation $x = \{x_i\}_{i=1}^I$ is* Pareto Better *than (or* Pareto Superior *to or* Pareto Improves *on) an allocation $y = \{y_i\}_{i=1}^I$ if it makes everybody at least weakly happier, and one person strictly so. I.e.*

$$
\begin{aligned}
u_i(x) &\geq u_i(y) \ \text{for all } i \\
u_j(x) &> u_j(y) \ \text{for some } j
\end{aligned}
$$

Notice a critical difference between this definition and the alternative one which says:

$$
\begin{aligned}
u_i(x_i) &\geq u_i(y_i) \ \text{for all } i \\
u_j(x_j) &> u_j(y_j) \ \text{for some } j
\end{aligned}
$$

in the former one we are allowing people to care about what others consume. This is on purpose, it is rare that the second set of conditions hold.

Definition 2 *An allocation $\{x_i\}_{i=1}^n$ is* Pareto Efficient *if there is no feasible Pareto better allocation*

Definition 3 *An x is* Pareto Efficient *if there is no Pareto better outcome. I.e. there does not exist a y*

$$
\begin{aligned}
u_i(x) &\leq u_i(y) \ \text{for all } i \\
u_j(x) &< u_j(y) \ \text{for some } j
\end{aligned}
$$

Notice an intriguing point about the definition. If we are at a Pareto efficient point, and we go to a different one, is the change Pareto efficient? No, it is not. You can only *Pareto improve* on a point to reach a Pareto efficient point, you can not Pareto improve on a Pareto efficient point, so transitioning between Pareto efficient points is not Pareto efficient.

Example 1 *Sharing a cookie.*
There are two people who are deciding how to split a cookie. Now I think it's fairly safe to say that we both consider getting more of the cookie to getting less. If this is true, then any division of the cookie is Pareto Efficient. Think about it, think about the definition. Everything that will make you happier (more of the cookie) will make me less happy. Thus everything is Pareto Efficient. What is not Pareto Efficient? Throwing some of the cookie away. Thus the reason people call it "Efficiency." Pure waste is never Pareto Efficient (as long as no one get's a kick from throwing away some of the cookie.)
Graphically, let's put the amount of the cookie for you on the vertical axis in a two dimensional graph, the

amount for me on the horizontal axis. Then we get something like the figure below.

What is efficient? Anything that is on the boundary, where (your share)=1-(my share).

Thus you can see that Pareto Efficiency is a very *weak* definition of what is good. But it also leads to some conclusions that are just distasteful.

Example 2 *The Sultan and the Saint*

The Sultan of Dubba-Dubba has three tigers. These tigers are very well fed, very well cared for. In fact—because it adds an extra lustre to their coats—the sultan gives them each four gallons of milk a day. Mother Teresa visits Dubba-Dubba and says, "Sultan of Dubba-Dubba, this is an immoral act. If you give me the milk, I can feed twelve babies who would starve to death without it."

Now, unfortunately, the Sultan studied economics in college, and because it suits his political agenda he equates morality with Pareto Efficiency. And he thinks about it and says to Mother Teresa: "Well, now let's analyze this carefully. For all practical purposes we can think of only you and me as being the people involved. The possible outcomes are T for feed the tigers, and B for feed the babies. Now I will make a graph with us listed in order across the top and the possible outcomes listed below us from best to worst for us individually:

Sultan	Mother Teresa
T	B
B	T

and as you can see, both are Pareto Efficient. To move from T to B as the state of the world would make me a little less happy. Thus we can't do it, and since I have the milk, you can go back to Calcutta."

Yuck, but unfortunately his methodology is correct. Want to think about something even worse? Let's say that Mother Teresa went to the UN, and said, "I'd like you to impose the tithe on all the people of the world so we can feed and care for the babies." For this example, imagine that the UN had more power than it currently does and for your information the tithe is a rule from the Bible that everyone should donate ten percent of their income to charitable causes. The sultan would be able to go to the UN and say that this solution was *not* Pareto Efficient. It would be Pareto Efficient to feed the babies, but not by means of a tithe.

So why do economists care about Pareto Efficiency when it has such unpleasant consequences? Well, for that we have to understand a little bit of history, and a little bit about Ordinality.

0.1 The Origin of Pareto Efficiency.

Pareto was an Italian economist around the turn of the century. He noticed a very fundamental and undeniable fact.

1. From market behavior, we can not tell the *level* of happiness people get from what they do. We can only whether they prefer one basket of goods (allocation) to another, or their *relative* preferences.

   This insight is exactly that *preferences are ordinal.* Then he began thinking about what this meant. Can you compare the happiness of two people with ordinal preferences? No, if their preferences are truly ordinal, the utility function $u(x)$ is equivalent to the utility function $10,000,000u(x)$, so how do we compare it with some other person's utility? Does–for example–

   $$\max_{x_1,x_2} u_1(x_1) + u_2(x_2)$$

   make sense? No, since utility functions one and two are ordinal—not cardinal—this is the same as

   $$\max_{x_1,x_2} 10,000,000u_1(x_1) + \frac{u_2(x_2)}{10,000}$$

   so Pareto concluded

2. Since preferences are ordinal; we can not compare two people's happiness.

   Thus what is left? The only "better than" or "worst than" statement you can make is

3. A is better than B only if A makes *everybody* happier.

In the in the maximization problem above, this means if

$$\frac{\partial u_1}{\partial z} > 0, \frac{\partial u_2}{\partial z} > 0$$

for some change in the considered allocation z, then it doesn't matter what ordinal translation we apply to u_1 and u_2, these will still both be positive. (To be precise, only one has to be strictly positive to be a Pareto improvement, if the other is zero that's OK). In the share-the-cookie example this is why efficiency implies no waste. More cookie makes us happier, so if there is some cookie left over we can give it to someone and make the world a better place.

And this is exactly what *Pareto Superior* means. Now where is the error in this logic? It's embedded between steps one and two. Economist generally have come to assume that since the *market* only revealed ordinal preferences that peoples preferences *were* ordinal. This is clearly not correct. But on the other hand, maximizing the sum of cardinal utility functions doesn't always make sense either. For one thing, it means that you take from the skillful and give to the unskilled. Or Marx's "from each according to his ability, to each according to his need," an amusing philosophical desideratum, but I'm not sure it's what we should mean by welfare either.

0.2 A "How To" manual.

Now I would hate to think that the Sultan learned about Pareto Efficiency from me, but his analytical method is very good. It can be generalized as follows:

1. List the people who's preferences would be significantly affected by the outcome $I = (1, 2, 3, ...)$

2. List the possible outcomes of the problem under discussion $\Omega = (A, B, C, ...)$

3. Put the people across the top of a chart, and then list the outcomes from best to worst below them.

I'll whip up a three person example right now. I'm doing this randomly, it's easy enough to do.

1	2	3
A	D	C
B	C	D
C	E	B
D	B	A
E	A	E

The people are 1,2, and 3. 1 prefers A to B to C to D to E, 2 prefers D to C to E to B to A.

Now, how do we find what is Pareto Efficient? The following is a convenient Algorithm.

Algorithm 1 $T_0 = \Omega, W_0 = \emptyset, P_0 = \emptyset$, continue the algorithm until $T_t = \emptyset$

1. *The first item in any person's column is Pareto Efficient, for example above* $P_1 = \{A, D, C\}$ $T_1 = T_0 \backslash P_1, W_1 = \emptyset$

t. *Select one of the items with highest rank from* T_{t-1}, *say* $\omega \in T_{t-1}$, *if there is anything which is better for everyone in* P_{t-1} *then* $W_t = W_{t-1} \cup \omega, P_t = P_{t-1}, T_t = T_{t-1} \backslash \omega$, *else* $W_t = W_{t-1}, P_t = P_{t-1} \cup \omega, T_t = T_{t-1} \backslash \omega$.

   When $T_t = \emptyset$ P_t *is the set of Pareto Efficient Allocations,* W_t *is the set of not Pareto Efficient allocations.*

So in the iterative step we must first consider B, the second item for person 1. A is the only item higher for 1, but for person 2, B is better than A, thus $B \in P_2$ and $T_2 = E$.

Thus the final step is to check E, and it's the last so let's *hope* it's not Pareto Efficient. The person for whom it's highest is 2, so we have to check $\{C, D\}$. Now for person 1, both C and D are better than E, likewise for person 3, thus E $W_3 = E$. $T_3 = \emptyset$.

So the Pareto Efficient outcomes are $\{A, B, C, D\}$. I suggest you do this repetitively until you understand how in your bones. It could be useful for tests, and you will begin to understand on an intuitive level what Pareto Efficiency is. From considering this algorithm we can realize two characteristics of Pareto Efficiency.

1. *Consensus decision making.* At all times, something that is not Pareto Efficient could be changed by consensus.

   E is not Pareto Efficient because everyone would agree to change to C.

2. *Tyranny of the Individual.* If one person is happy, no matter how unhappy the other people are it's still Pareto Efficient.

   A makes persons 2 and 3 very unhappy. For 2 it is the worst thing, and for 3 it is the next to worst. But, it makes 1 happy, so it's Efficient. The Sultan and the Saint is another example of this tyranny. I think most people would agree that Mother Teresa should get the milk, but the Sultan's trivial preferences for silky coats on his tigers outweighs them all. Another example: say that someone is happy going around shooting people, is it Pareto Efficient to let him slaughter everyone in Ankara? As long as every killing makes him a little happier, yes.

0.3   A Critique of Pareto Efficiency and The Competitive Market.

This is primarily based on the work of Amartya Sen. While he did when a Nobel Prize it was not for his work on understanding Pareto Efficiency. A pity, I think it is his greatest contribution. One of his earliest contributions on this subject was "The Impossibility of a Paretian Liberal." in the Journal of Political Economy, Vol. 78, No. 1. (Jan. - Feb., 1970), pp. 152-157.

One argument in the defence of the competitive market—and therefore Pareto efficiency, it's most exciting outcome—is that it is as good as the economy can do without violating people's "fundamental" right to property. It is the best we can do with *voluntary* trades. Well that's interesting and viable. Unfortunately it's not exactly true. One of the fundamental conditions of a definition of property rights is the right of free disposal. If I don't want it I can choose to throw it away.

Definition 4 A person has the right of free disposal *if given any allocation $\{x_i\}$ they can consume any* $0 \leq\leq \hat{x}_i \leq\leq x_i$

We always assume this, in fact it seems in some ways a necessary extension of the right to do what you will with your property—or trade. But consider the following example.

Example 3 *Harry and his Grandpa at the Hamburger Stand.*

Harry and his Grandpa are both very concerned about their health. They both agree they should avoid fatty foods, and both of them are extremely concerned about Grandpa in particular. He's already had one heart attack and has been warned to avoid fat if he wants to live. They differ in one way, however. Grandpa was a child of the great depression, and so would never waste any food. Harry on the other hand has never known want, so he thinks it's better to waste food than eat fatty food.

Now Grandpa is given a nice, juicy, greasy heart attack special hamburger by the hamburger vendor (who doesn't particularly like either Harry or his Grandpa.)

What happens now?

Outcomes:

1. *Harry eats the hamburger* (HH)

2. *Grandpa eats the hamburger* (GH)

3. *The hamburger is wasted* (WH)

And their preferences are:

H	G
WH	HH
HH	GH
GH	WH

The efficient outcomes are $\{HH, WH\}$, but under the right of free disposal, $\{GH\}$ will be the outcome. Why? If Grandpa gives the hamburger to Harry, then he will throw it away as is his fundamental right. Thus he will consume the hamburger himself since he prefers $\{GH\}$ to $\{WH\}$.

Disturbing, ehh? He calls what I call the right of free disposal "libertarian rights" in general. The right of free speech, the right to self defense, anything can be used to whip up this kind of example. (His example was the right to free speech—to read whatever you want.)

Notice that this is why if x is an allocation, we generally assume that $u_i(x) = u_i(x_i)$, or people don't care about other peoples outcomes. Clearly a false assumption, and thus in general pareto efficiency conflicts with all sorts of things. If u_i is a non-degenerate function of x_j then this is—in point of fact—an externality. There has to be a separate market (which would hardly be competitive) over what j consumes. "Personalized prices" where i buys some of what j wants to consume.

Some other points Sen makes in "Markets and Freedom: Achievements and Limitations of the Market Mechanism in Promoting Individual Freedoms." *Oxford Economic Papers* 45:519-541 are:

1. Producer has a "right to the surplus of his produce" is equivalent to Marxian "labor theory of value." The market is allocation mechanism, not justice. You don't "earn what you deserve."

   The diamond/water paradox: Consider how much more valuable water is than diamonds. Yet the price of one diamond is as much as the cost of years worth of water, which can be essentially free. Why is this? Adam Smith pointed out that it was because of *marginal cost pricing*. Very accurate deduction, and very astute. But what does it imply? Total revenues has absolutely no relation to total benefit.

   To give you another example, consider the two graphs below. In both markets the price is 10, the quantity 5, yet the consumer surplus in the left hand market is much greater than that in the right hand market.

Thus it is obvious that the price in the two markets has nothing to do with the total value of the two goods.
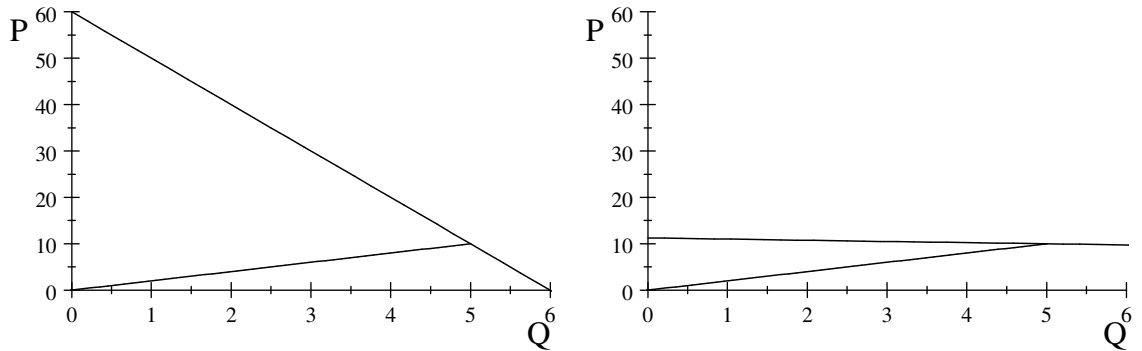
Note this indicates that there is no reason to assume that just because someone earns more, they are more valuable to society. And therefore the benefit of a professor is not necessarily less than a wall street analyst. I love to think about that at night.

2. Rights based argument: Many famines have been shown to be caused by people exercising their legally defined property rights. "Sudden shifts in entitlements." Does it make sense to put such priorities on means when the end can be so horrible?

3. Second Welfare Theorem is the tool of revolutionaries. Those who wish to interfere in the market mechanism.

   Of course you can achieve any pareto efficient outcome, but have to do it by revolution. Notice what this highlights, *for the properly defined property rights you can achieve anything.* Or *there are many market equilibria, we must know the social system (property rights) to know which one is selected.*

 Some arguments I would like to add:

1. Do you know what the pareto efficient treatment for cancer is? Leave them cancer cells alone, they're happy!

   Why should the society, as an organism, treat it's cells—human beings—as if their utility was paramount to it's utility? Only economists—who implicitly worship the individual in case you haven't noticed—would be so silly as to suggest this.

2. Do you know why economists have spoiled children? It's not pareto improving to discipline them.

   No, seriously, I mean this. Take a classic case. A child is having a lot of fun, and the mother says "you have to go to bed now." The mother will argue that "tomorrow morning you will be grumpy if you don't go to sleep now." But the child—being a rational actor—already knows this.

   Or forget that they might know this, say they respond, "I know." Are you going to just let them stay up? No, I won't, and I doubt you would. You will send them to bed, even though it is strictly worse for them, and thus is not a pareto improvement. Assuming you were at a pareto efficient state, this is pareto worse.

3. There are multiple equilibria, the second welfare theorem proves it's just a matter of defining property rights. Thus we can select any equilibrium we want. The market (to a revolutionary) places no restrictions on which pareto efficient outcome is preferred. So why do all this analysis based on the "current property rights?"

But let me give respect where respect is due. And I think this is something Amartya Sen would agree with. If you have an engineering background imagine the following problem (if you don't find a friend who does.)

You have 67 million people, and $400 billion worth of goods. You have to allocate these goods in some what that no one will want to trade with anyone else afterwards.

Are you frightened? If not you haven't really thought this through. It is fundamentally terrifying. And yet this is what the market does every day. Every year. Every century. Can you do better?

And you should think about that thought carefully. As I've said before I am a radical positivist. I am terrified to start modifying the economy before we really understand it. I am much more comfortable studying and understanding what is, then trying to figure out how it should be, and how we should change it.

Why? Can I say "Soviet Union"? Why did it fail? Corruption (or powerful people lining their pockets)? That's rampant in many economies. The arms race? Important, no doubt, but the United States' Economy withstood it—and there's lots of corruption there as well. What is unique in the Soviet Block countries? A government that tried to price "from each according to their ability, to each according to his need." They tried to interfere with the market mechanism. Notice that the communist Chinese appear to agree with me. They are introducing a market economy—they still have their corruption, still have their large army. Slowly, and hopefully with justice and kindness on their front burner, but they are doing it. Quite flying in the face of Marx's dream, ehh? I am not using this to argue against Marxism, I don't understand what he said well enough to do that. I am pointing out how hard it is to try to centrally manage an economy.

At the same time let me point out that no competitive economy can exist without some government regulation. The natural reaction to a competitor is to kill her. No better way to protect your market. And don't forget that innovation, etcetera, depends on government regulated monopoly power. There is no such as the "free market" but it seems that governments who try to intervene too much fail. The market is a naturally occurring organism, it will fight back if you try to control it too tightly.

It is hard to intervene, necessary but hard. Sometimes the best intentions and theories can lead to the worst disasters. Often it's best to just let it be. The market economy is in some ways horrible, but I don't know what to replace it with.