The Welfare Lecture.

April 11, 2014
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What is welfare? I only wish I knew. Unfortunately this is one of those areas where people have many different definitions, and the definitions often contradict each other. Is *fairness* important? Sure, seems it should be. What type of fairness? Should I buy a Barbie doll for my son whenever I buy one for my daughter? And what if we have two people, one of whom who works very hard and the other one is lazy. Would it be fair for me to pay them the same amount? What about the communist concept of "from each according to his ability, to each according to his need"? Justice, freedom, and rights? What are these concepts and how do we apply them? And not to mention that we also need to decide whether we are going to build it based on utility. This is not a trivial issue, in economics we take preferences as fixed but they do change over time and might imply something we don't want. If someone wants to shoot up heroin until it kills them should society let them? I wouldn't.

But hey, maybe we could take a simple and obvious idea and apply that. How about we change what we are doing if there is something else that makes everyone happier (Pareto dominance)? Gosh, that's a great idea. We can go far with that. This is Pareto efficiency, and undeniably a good concept. On the other hand this concept will come into conflict with all of the ideas above. In economics this makes those idea "bad" or only "good to the extent we can also respect Pareto efficiency." That is bad.

Not that I'm a big fan of the other approach in economics, just simply sum up everyone's utility and maximize. This often does imply "from each according to his ability, to each according to his need." You may like this idea, but realize it means that the hard working are going to give to the lazy. What is even more egregious is people who claim they are analyzing Pareto efficiency and then just maximize the sum of utilities.

Throughout these notes I will often have to explain different terms for concepts. Like any language different people use different terms for the same thing, and to avoid confusion I need to introduce them all.

1 What is Pareto Efficiency?

First we need to define the things we are going to choose over. These things are called (formally) *allocations* or *states of the world,* or sometimes (informally) *outcomes* or *options*. It is very important that you understand what kind of things we are going to choose over, so let me give a precise definition of an allocation.

Definition 1 *An* allocation *is a complete list of what every person in the society receives. These will usually be denoted by capital letters:* $\{A, B, C, D, E, ...\}$

To give a counter example saying "I get 10,000 TL" is not an allocation, in an allocation (at least) you need to tell me the amount of money each person in society is going to receive.

Let's make this clear with a concrete example. Let's divide a cookie, and assume that both people would like to eat the whole thing. Then an allocation is a share of the cookie for our two people, (s_1, s_2), and it has to be feasible, so $0 \leq s_1 \leq 1$, $0 \leq s_2 \leq 1$ and $s_1 + s_2 \leq 1$. Now some simple allocations might be $A = \left(\frac{1}{4}, \frac{1}{2}\right)$ and $B = \left(\frac{1}{5}, \frac{3}{4}\right)$, neither of these allocations particularly make sense because they both waste cookie, but who cares? Now you might think "1 can get A $(\frac{1}{4})$ and 2 can get B $(\frac{3}{4})$" and yes, that is feasible. BUT this is a third allocation, let's call it $C = \left(\frac{1}{4}, \frac{3}{4}\right)$. When we are choosing between A and B we can only choose to give 1 $\frac{1}{4}$ and 2 $\frac{1}{2}$ (A) or 1 $\frac{1}{5}$ and 2 $\frac{3}{4}$ (B). For arbitrary A and B we can not just magically create a new allocation to choose where some people get what they would in A and the rest get what they would in B. If this is an option then, like in our example, we need to list it as another allocation.

Definition 2 *An allocation (let's call it A)* Pareto Dominates *another allocation (let's call it B) if everyone prefers A to B and one person strictly prefers A to B.*

Again this is one of those terms that is used so frequently that there are many different words that are used for the same concept. Pareto Improvement, Pareto Better, Pareto Superior, and probably more. I like Pareto dominance because if one thing dominates another it is always better. Here "always" refers to "every person in society." But I don't have strong feelings about it.

The statement "one person strictly prefers" is just a technical way of saying that not everybody can be indifferent. If everyone is indifferent between A and B then surely we don't want A to be Pareto dominant. At least one person must strictly want A more, and then that makes it a Pareto improvement. Notice it only takes one, though of course it can be more.

Pareto dominance is just a way of ordering the allocations. You can see that it's fairly innocuous on it's surface. If everyone wants to change to A, then we do it. Who could object? Certainly not me, unless you were going to say that it's the *only* important thing.

Definition 3 (1) *A state of the world A is* Pareto Efficient *if it isn't Pareto Dominated.*

The problem I always find with this definition is it defines what *is* Pareto efficient by what is *not*. A very confusing way to proceed. Instead of describing what is Pareto efficient in a positive manner, it tells you how to find out what is not, then says whatever's left over is Pareto efficient. So let's try another definition.

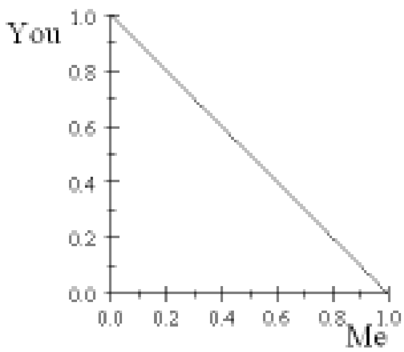
Definition 4 (2) *A state of the world is* Pareto Efficient *if everything that makes somebody (strictly) happier makes someone else (strictly) less happy.*

Does that make you happier? Let's go into a few examples to make this clearer.

Example 5 *Sharing a cookie.*

So to go back to my first example of the states of the world. There are two people who are deciding how to split a cookie. Now I think it's fairly safe to say that we both consider getting more of the cookie to getting less. If this is true, then any division of the cookie is Pareto efficient. Think about it, think about the definition. Everything that will make you happier (more of the cookie) will make me less happy. Thus everything is Pareto efficient. What is not Pareto efficient? Throwing some of the cookie away. Thus the reason people call it "efficiency." Pure waste is never Pareto efficient (as long as no one get's a kick from throwing away some of the cookie.)

Graphically, let's put the amount of the cookie for you on the vertical axis in a two dimensional graph, the amount for me on the horizontal axis. Then we get something like the figure below.



What is efficient? Anything that is on the boundary, where (your share)=1-(my share).

Some other professors will use *Pareto Optimal* for this concept, but this example I think makes clear why I prefer the term *Pareto efficiency.* One of the definitions of *efficient*[1] is "productive without waste." In this example what is Pareto efficient is outcomes that have no *social* waste. There is no sense in which we are finding things that are *optimal* (most desirable or satisfactory)[2] ask my Mom, she would tell you that the optimal thing is quite clearly to split the cookie fairly. And indeed, as the next example shows Pareto efficient outcomes can sometimes be quite undesirable and unsatisfactory.

Example 6 *The Sultan and the Saint*

The Sultan of Dubba-Dubba has three tigers. These tigers are very well fed, very well cared for. In fact—because it adds an extra lustre to their coats—the

[1] http://www.merriam-webster.com/dictionary/efficient
[2] http://www.merriam-webster.com/dictionary/optimal

sultan gives them each four gallons of milk a day. Mother Teresa visits Dubba-Dubba and says, "Sultan of Dubba-Dubba, this is an immoral act. If you give me the milk, I can feed twelve babies who would starve to death without it."

Now, unfortunately, the Sultan studied economics in college, and because it suits his political agenda he equates morality with Pareto efficiency. And he thinks about it and says to Mother Teresa: "Well, now let's analyze this carefully. For all practical purposes we can think of only you and me as being the people involved. The possible outcomes are T for feed the tigers, and B for feed the babies. Now I will make a table with us listed in order across the top and the possible outcomes listed below us from best to worst for us individually:

Sultan	*Mother Teresa*
T	*B*
B	*T*

and as you can see, both are Pareto efficient. To move from T to B as the state of the world would make me a little less happy. Thus we can't do it, and since I have the milk, you can go back to Calcutta."

Yuck, but unfortunately his methodology is correct. Want to think about something even worse? Let's say that Mother Teresa went to the UN, and said, "I'd like you to impose the tithe on all the people of the world so we can feed and care for the babies." For this example, imagine that the UN has more power than it currently does and for your information the tithe is a rule from the Bible that everyone should donate ten percent of their income to charitable causes. The sultan would be able to go to the UN and say that this solution was *not* Pareto efficient. It would be Pareto efficient to feed the babies, but not by means of a tithe.

So why do economists care about Pareto efficiency when it has such unpleasant consequences? Well, for that we have to understand a little bit about the science of economics and ordinality.

1.1 Why Economists think Pareto efficiency is so important.

Economics is based on the rule of *revealed preferences.* The argument is essentially that people will *say* they want to do a lot of things, but what do they actually do? What they actually **do** tells us how they really feel, they might say a lot of different things for a lot of different reasons. Unfortunately market decisions do not tell us enough about preferences. From market behavior, we can not tell the *level* of happiness people get from what they do. We can only whether they prefer one basket of goods (allocation) to another, or their *relative* preferences. This insight is exactly that:

Conclusion 7 *Market behavior only reveals ordinal preferences.*

To explain it say that I see you choosing the (vector) of options X when you could have had Y (by vector I mean this is a whole bunch of goods and services, not just one or two.) This tells us that

$$u(X) \geq u(Y)$$

but consider any arbitrary function $f(x)$ which is increasing ($f' > 0$ or $x > y$ implies $f(x) > f(y)$), then it's also obvious that

$$f(u(X)) \geq f(u(Y)) \ .$$

Now your level of happiness is $u(X)$, but we have just shown is that it could also be $f(u(X))$. To put this in simple English, some people are always depressed no matter what you give them, they could be King of the world and still be depressed. Other people are happy whenever they see food on the table, or if you just say hello to them. Some simple examples of ordinal transformations? $f(x) = \frac{x^{1-\sigma}}{1-\sigma}$ ($\sigma \neq 1$), $f(x) = \ln x$, or the most important one right now: $f(x) = \alpha u(x) + \beta$ where $\alpha > 0$ and β can be any real number.[3]

Now let us think about what this means for some sort of "maximizing people's happiness." Let's assume that $u(x) = \ln x$ and that we're sharing a cookie. Our objective might be:

$$\max_{(s_1,s_2)} \ln s_1 + \ln s_2 \text{ such that } s_1 + s_2 \leq 1 \ .$$

But since utility is ordinal the solution must be the same when we multiply person 1's utility by α and person 2's utility by $2 - \alpha$ (where $0 \leq \alpha \leq 2$) or:

$$\max_{(s_1,s_2)} \alpha \ln s_1 + (2-\alpha) \ln s_2 \text{ such that } s_1 + s_2 \leq 1 \ .$$

You probably know how to solve this. First of all we can just assume that $s_2 = 1 - s_1$ and

$$\max_{s_1} \alpha \ln s_1 + (2-\alpha) \ln (1 - s_1)$$

$$\begin{aligned}
\frac{\alpha}{s_1} - \frac{2-\alpha}{1-s_1} &= 0 \\
\frac{\alpha}{s_1} &= \frac{2-\alpha}{1-s_1} \\
\alpha (1 - s_1) &= s_1 (2 - \alpha) \\
\alpha &= s_1 (2 - \alpha + \alpha) \\
s_1 &= \frac{\alpha}{2}
\end{aligned}$$

OK, so the solution depends on α, but what does this mean? Since we don't know the "true" value of α (indeed we're not sure it exists) it means:

[3] If we include uncertainty in the model the last one is all that we can do, but it is enough.

Conclusion 8 *We can not compare the happiness of different people.*

If we could then we could tell you the optimal way to split the cookie for every α, but we can't.

What could make both people happier for *every* value of α. Well consider a small local change, from (s_1, s_2) to $(s_1 + \lambda_1\Delta, s_2 + \lambda_2\Delta)$ for small Δ this is just the equivalent of taking the derivative of the welfare function

$$W(s_1 + \lambda_1\Delta, s_2 + \lambda_2\Delta) = \alpha \ln(s_1 + \lambda_1\Delta) + (2 - \alpha)\ln(s_2 + \lambda_2\Delta)$$

with respect to Δ:

$$\lambda_1 \frac{\alpha}{s_1 + \lambda_1\Delta} + \lambda_2 \frac{2 - \alpha}{s_2 + \lambda_2\Delta}$$

and we want this derivative to be non-negative for every $\alpha \in [0, 2]$, but this implies $0 \leq \min(\lambda_1, \lambda_2)$ and $0 < \max(\lambda_1, \lambda_2)$ or:

Conclusion 9 *A is better than B if it makes at least one person better off without hurting anyone else.*

And this is the definition of Pareto dominance.

Now where is the error in this logic? It's embedded between the first and second conclusion. Just because the *market* only reveals ordinal preferences doesn't mean preferences *are* ordinal. All we need to reveal cardinal preferences is to have someone choose between two people's happiness. We do this all the time. Parents do it and so do Governments.

2 A "How To" manual.

Now I would hate to think that the Sultan learned about Pareto efficiency from me, but his method is very good. It can be generalized as follows:

1. List the people who's preferences would be significantly affected by the outcome (1,2,3,...)

2. List the possible outcomes of the problem under discussion (A,B,C,...)

3. Put the people across the top of a chart, and then list the outcomes from best to worst below them.

I'll whip up a three person example right now. I'm doing this randomly, it's easy enough to do.

1	2	3
A	*D*	*C*
B	*C*	*D*
C	*E*	*B*
D	*B*	*A*
E	*A*	*E*

The people are 1,2, and 3. 1 prefers A to B to C to D to E, 2 prefers D to C to E to B to A.

Now, how do we find what is Pareto efficient? I'm going to explain two different techniques. One of which is a standard formal methodology and the other of which is a better version of the same methodology.

To define the formal methodology let $P_i(X) = \{\text{Things } i \text{ thinks are strictly better than } X\}$— notice that X is **not** better than X. Then X is Pareto efficient if $\emptyset = \cap_{i \in I} P_i(X)$, if it is not empty then those things Pareto dominate it. So let me use this technique in this economy.

$P_1(A) = \emptyset$ | $P_1(B) = \{A\}$ | $P_1(C) = \{A, B\}$
$P_2(A) = \{B, C, D, E\}$ | $P_2(B) = \{C, D, E\}$ | $P_2(C) = \{D\}$
$P_3(A) = \{B, C, D\}$ | $P_3(B) = \{C, D\}$ | $P_3(C) = \emptyset$
$\cap_{i=1,2,3} P_i(A) = \emptyset$ | $\cap_{i=1,2,3} P_i(B) = \emptyset$ | $\cap_{i=1,2,3} P_i(C) = \emptyset$
A is P.E. | B is P.E. | C is P.E.

$P_1(D) = \{A, B, C\}$ | $P_1(E) = \{A, B, C, D\}$
$P_2(D) = \emptyset$ | $P_2(E) = \{C, D\}$
$P_3(D) = \{C\}$ | $P_3(E) = \{A, B, C, D\}$
$\cap_{i=1,2,3} P_i(D) = \emptyset$ | $\cap_{i=1,2,3} P_i(E) = \{C, D\}$
D is P.E. | E is P. dominated by C, D

Now that took a long time. Is there any way we can do it more quickly? Well first of all we notice that if X is top ranked for anyone then it must be Pareto efficient. Then we notice that if we start with high ranked things they'll be easier to check. Let Ω be all the allocations, then a more efficient way of solving the problem is:

1. The top item in any person's column is Pareto efficient. Let PE_0 be the set of these allocations, $U_0 = \Omega \backslash PE_0$, $W_0 = \emptyset$.

   In our example, $PE_0 = \{A, C, D\}$, $U_0 = \{B, E\}$, and $W_0 = \emptyset$. Now we go to the iterative step.

t Go to one of the highest ranked allocations (in someone's preferences) in U_{t-1}, call it X. Find if something this person prefers to X are also preferred by the other people in the economy. If not then $PE_t = PE_{t-1} \cup X$, $U_t = U_{t-1} \backslash X$, $W_t = W_{t-1}$. If yes $W_t = W_{t-1} \cup X$, $U_t = U_{t-1} \backslash X$, $PE_t = PE_{t-1}$.

And you continue this algorithm until $U_t = \emptyset$.

So let me show you how you do this iterative step. B is second ranked in person 1's preferences, and $P_1(B) = \{A\}$. Now instead of finding $P_2(B)$ and $P_3(B)$ I just check whether A is better than B for these other two people. It is not, person 2 likes B better than A (which is the same as saying that $P_1(B) \cap P_2(B) = \emptyset$) and so $PE_1 = PE_0 \cup B = \{A, B, C, D\}$ ($U_1 = \{E\}$, $W_1 = \emptyset$).

Now E is third highest for 2, $P_2(E) = \{C, D\}$. Does 1 like either C or D better than E? Yes, she likes them both better. What about person 3? He ranks E the worst, so he likes both C and D better. So $W_2 = \{E\}$ $PE_2 = \{A, B, C, D\}$, $U_2 = \emptyset$. We are done.

I personally think the second method is easier and faster. Not just because you can accept a lot of things in the first step but because the number of comparisons you do at each step is smaller. Of course you can use either method, or come up with your own. Anyway you slice it the Pareto efficient outcomes are $\{A, B, C, D\}$. I suggest you do this repetitively until you understand how in your bones. Learning how to do this will help you understand on an intuitive level what Pareto efficiency is. From considering this algorithm we can recognize two characteristics of Pareto efficiency.

1. *Consensus decision making.* At all times, something that is not Pareto efficient could be changed by everyone agreeing.

   E is not Pareto efficient because everyone would agree to change to C. Of course the flip side of consensus is:

2. *Tyranny of the Individual.* If one person is happy, no matter how unhappy the other people are it's still Pareto efficient.

A makes persons 2 and 3 very unhappy. For 2 it is the worst thing, and for 3 it is the next to worst. But, it makes 1 happy, so it's efficient. The Sultan and the Saint is another example of this tyranny. I think most people would agree that Mother Teresa should get the milk, but the Sultan's trivial preferences for silky coats on his tigers outweighs them all. Another example: say that someone is happy going around shooting people, is it Pareto efficient to let him slaughter everyone in Ankara? As long as every killing makes him or her a little happier, yes.

So let me give you two other examples. One of which illustrates a useful technique that can *sometimes* make it easier to find Pareto efficient allocations, and the other which shows it won't always work.

1	2	3
$\underline{A}$	B	$\underline{A}$
B	$\underline{A}$	D
C	F	C
D	E	F
E	C	E
F	D	B

In this economy its obvious that A and B are Pareto efficient, but what else? Well if we draw a line under the A allocation we see that everything else is always lower than that line. Thus A Pareto dominates them all.

But here's an example where this absolutely doesn't work.

1	2	3
A	C	F
B	D	A
C	F	B
D	A	C
E	B	D

You can check that in this economy the Pareto efficient are $\{A, C, F\}$. F does not Pareto dominate anything; A Pareto dominates B; and C Pareto dominates D. Each Pareto efficient option dominates at most one thing, so drawing lines doesn't help much.

2.1 Some Interesting Comparative Statics about Pareto Efficiency

So I want to point out some interesting comparative statics about Pareto efficiency, in other words how things change when we change something about the problem. Consider the following three economies:

1	2	3
A	A	A
B	B	B
C	C	C
D	D	D
E	E	E
F	F	F
	α	

1	2	3
A	F	E
B	A	F
C	B	A
D	C	B
E	D	C
F	E	D
	β	

1	2	3
A	A	F
B	B	E
C	C	D
D	D	C
E	E	B
F	F	A
	δ	

Now what is Pareto efficient in economy α? Obviously only A, why? Because everyone has the same preferences. What is Pareto efficient in economy β? Well now they disagree about what is best, so $\{A, E, F\}$ are all Pareto efficient. But notice that on $\{A, B, C, D\}$ they have the same preferences, so A Pareto dominates $\{B, C, D\}$. Economy δ? Everything. Yes, you read right, everything. Why? Because person 3 (and only person 3) completely disagrees on the order between any two options with persons 1 and 2. If 1 likes X better than Y then 3 likes Y better than X.

Conclusion 10 *The more different people's preferences the more options will be Pareto efficient.*

Now let's consider adding a person to economy β. What is going to happen to the number of Pareto efficient things? Well you should be able to figure out the answer from the last conclusion. There must be more disagreement in the new economy so there will be more Pareto efficient things. This new person (in the best case) can have exactly the same preferences as one of the people already in the economy. In the worst case she will have exactly the opposite preferences. Intermediate? Well that's the third example below:

1	2	3	4
A	F	E	A
B	A	F	B
C	B	A	C
D	C	B	D
E	D	C	E
F	E	D	F

$\beta\alpha$

1	2	3	4
A	F	E	F
B	A	F	E
C	B	A	D
D	C	B	C
E	D	C	B
F	E	D	A

$\beta\beta$

1	2	3	4
A	F	E	D
B	A	F	E
C	B	A	F
D	C	B	A
E	D	C	B
F	E	D	C

$\beta\delta$

Economy $\beta\alpha$ has exactly the same Pareto efficient allocations as β, because the new person completely agrees with person 1. In economy $\beta\beta$ everything is Pareto efficient because the new person completely disagrees with person 1. In economy $\beta\delta$ we have one new Pareto efficient allocation, D. In all cases *of course* what was Pareto efficient before is still Pareto efficient. If we couldn't get n people to agree that something was better how can we get $n+1$ people to agree?

Conclusion 11 *As the number of people in society increases so does the number of Pareto efficient allocations.*

3 Pareto Efficiency in the Continuum

The standard practice is to introduce Pareto efficiency in an economy where all goods are continuous. In this case Pareto efficiency becomes a restriction on the first order conditions, which is actually easier to check. However it also makes it seem like a lot of things (shall we call them "stupid things") are not Pareto efficient. To be frank there is always a continuum of thing that are Pareto efficient—it's just so small relative to the total continuum.

Pareto efficiency requires *Productive Efficiency*, which is $MC_1 = MC_2$ for any firms 1 and 2 in the same industry. Why does it require this? Well assume that $MC_1 > MC_2$, and consider the following complicated contract. Firm 1 offers to buy Δ (small) units of output from firm 2 and pay them $\frac{1}{2}MC_1 + \frac{1}{2}MC_2$. The change in firm 1's costs will be $-MC_1\Delta + \left(\frac{1}{2}MC_1 + \frac{1}{2}MC_2\right)\Delta = \frac{1}{2}\Delta\left(MC_2 - MC_1\right) < 0$ and firm 2 will be willing to make this trade because $\left(\frac{1}{2}MC_1 + \frac{1}{2}MC_2\right)\Delta > \Delta MC_2$. Thus both firms benefit. But what does the competitive economy give us? $P = MC_1 = MC_2$. Cool, ehh?

Pareto efficiency also requires *Allocational Efficiency,* or:

$$\frac{MU_F^1}{MU_C^1} = \frac{MU_F^2}{MU_C^2}$$

where the marginal utility of person i with respect to good X is MU_X^i. Why? Well assume $\frac{MU_F^1}{MU_C^1} > \frac{MU_F^2}{MU_C^2}$. This means that person 1—relatively speaking—values F more than person 2 does. So what is the trade? Person one gives ΔC

to 2 and person 2 gives ΔF to one. Again, what does the competitive economy give us?

$$\frac{MU_F^1}{MU_C^1} = \frac{p_F}{p_C} = \frac{MU_F^2}{MU_C^2}$$

And this is perhaps what is most amazing about Pareto efficiency.

The perfectly competitive market is Pareto efficient. Truly amazing, and not a casual result. As you should recognize the more you study economics, it is one of the few situations where equilibrium is Pareto efficient. And much of the world does seem perfectly competitive. Not all of it, and not completely, but it does seem like a reasonable description much of the time.

There is another thing about Pareto efficiency, or about the market. The market can be thought of as a set of voluntary transactions. The perfectly competitive equilibrium is then the result when people engage in this type of trade. So trade can be thought of as exhausting all of the possible ways to make everyone happier. This is the first welfare theorem.

Theorem 12 *(First Welfare Theorem) Given people's initial incomes, the market exhausts all possibilities to make everyone happier, or the competitive marketplace is Pareto efficient.*

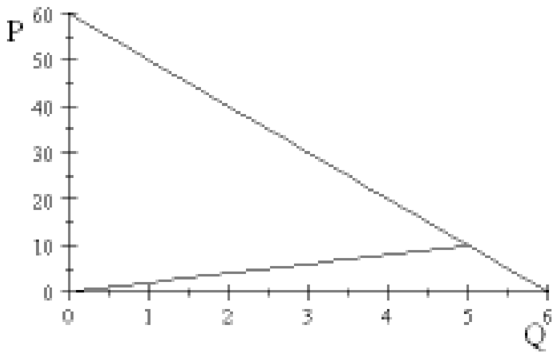
This is no small result, and notice something about this. In a world where income is initially distributed equally (and there is no prejudice, etceteras) the outcome will be in some sense "fair" in that all people will be happiest with their personal outcome. When they take into consideration the work they (and other people) do, no one will wish to trade shoes with anyone else. Pretty cool, ehh? Too bad incomes aren't equal, and that there is prejudice, and etceteras.

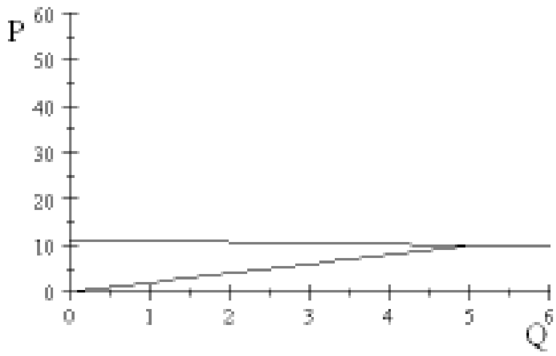
This is also Adam Smith's *invisible hand*. The marketplace is a place where it is fair to say people are mostly motivated by their selfish self interest. And yet, the market, like some invisible hand, will make sure that resources are allocated to where they have the most marginal benefit.

But let me also make another point. People like to think there is some diametric opposition between free markets (or "capitalism") and communism. A standard dictum of communism is "from each according to ability, to each according to his need." And people like to partially switch this maxim around and claim it holds under capitalism: "to each according to his ability." Or, put more simply, people "get what they deserve." Yea, right. This is true within an industry, but across industries? I wish.

When I hear the term "get what they deserve" I associate it with "they receive the value of their production." This is not at all true. This relates to another thing Adam Smith noticed, the *diamond-water paradox.* Diamonds–in total–have practically no value in your life. Sure, you like looking at them, and sure, you can resell them for lots of money, but they have such little intrinsic value. And water? It is life itself. So how much do we pay for water? Very little, its almost free. How much do we pay for diamonds?

To give you another example, consider the two graphs below. In both markets the price is 10, the quantity 5, the producer surplus 25, yet the consumer surplus in the left hand market is much greater than that in the right hand market. Thus it is obvious that the profits in the two markets has nothing to do with the total value of the two goods.





The final point? Pareto efficiency is an important characteristic of the competitive marketplace. It is also a desirable thing to do, and easy to measure. While welfare should not be defined as Pareto efficiency, welfare should consider Pareto efficiency.

4 Welfare in Economics

Welfare, defined as maximizing the sum of utilities, does have a place in economics. I want to show you why and when just to be exactly clear about what is required to turn Pareto efficiency into Welfare maximization. A proper characterization of Pareto efficiency is that there is some $(\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n)$ such that $0 \leq \alpha_i \leq n$ and $\alpha_1 + \alpha_2 + \alpha_3 + ... + \alpha_n = n$ and the allocation maximizes:

$$
\begin{aligned}
W(\alpha) &= \max_X \sum_{i=1}^{n} \alpha_i u_i(X_i) \\
&= \max_X \alpha_1 u_1(X_1) + \alpha_2 u_2(X_2) + \alpha_3 u_3(X_3) + ... + \alpha_n u_n(X_n)
\end{aligned}
$$

where X_i is the amount of resources person i is given and X is this vector for all people. (Yes, it's complicated, but just don't worry about it too much.) Now let us assume that utility is *quasi-linear* or $u_i(X_i) = v_i(X_i) + w_i$, where w_i is the amount of *wealth* this person is given. This is a very weird utility function, among other things it means that income has no impact on consumption. It is easy to show that the standard utility maximization problem is actually a "profit maximization" problem, or:

$$\max_{(X_i, w_i)} u_i(X_i) - \lambda(p' X_i - w_i) = \max_{X_i} v_i(X_i) - p' X_i$$

where p is the vector of prices. But what I want to show is that this implies that $\alpha_1 = \alpha_2 = \alpha_3 = ... = \alpha_n = 1$. To see this consider maximizing the welfare function

$$W(\alpha) = \max_{X,w} \sum_{i=1}^{n} \alpha_i (v_i(X_i) + w_i)$$

where $w = [w_1, w_2, w_3, ..., w_n]$ is the wealth allocation problem we need to solve. Now we have the constraint that $\sum_{i=1}^{n} w_i = w_0$, or that there is some initial wealth level (w_0) that we will allocate among the people. In other words, we are explicitly conducting transfers as needed. So in order to get rid of the constraint we can get rid of w_1 by writing $w_1 = w_0 - \sum_{i=2}^{n} w_i$. Now when we take the derivative with regards to w_i we get $\alpha_1 - \alpha_i = 0$ or $\alpha_1 = \alpha_i = 1$. Thus welfare maximization becomes:

$$W = \max_{X} \sum_{i=1}^{n} v_i(X_i) \ ,$$

or the maximization of the sum of cardinal utilities.

So why am I bothering to show you this? Well it's the formal basis of the standard welfare function we see:

$$W(Q) = CS(Q) + \Pi(Q)$$

where $CS(Q)$ is the consumer surplus, and $\Pi(Q)$ is the profit of the firms (notice these are a source of wealth for the consumers). But more importantly:

Conclusion 13 *Pareto efficiency is the same as Welfare maximization if* ***and only if*** *the welfare maximizer transfers wealth among individuals.*

Why does this matter? Well let's consider breaking up a monopoly—like Windows. Do you really think that society could not pay Microsoft enough for them to be willing to give up their source code? I doubt it; I'm sure the asking price would be in the trillions; but I doubt it.

But society does not do this, I'm fairly sure they never have when they've decided to break up a monopoly or a cartel. They just say "you're bad, stop." This is welfare increasing (under the assumptions above) but is **not** Pareto improving. So when you are doing welfare function analysis, be sure to remember that what is Welfare improving is not the same as what is Pareto improving. Pareto improving is a harder criterion, and Welfare improving is only equivalent in the ethereal model where the welfare maximizer *actually* transfers wealth between people.

Don't get me wrong. I think Windows should be made public property (at least an outdated version), and I don't think we really need to give Microsoft any more money. I just recognize that I don't think Pareto efficiency is the only proper welfare criterion.