

Definition 1 A game is one of perfect recall if

1. $\{x_i, x'_i\} \in h_i$ then x_i is neither a successor or predecessor of x'_i
2. If $\{x_i, x'_i\} \in h_i$ then any action that i knows about that leads to x_i must also lead to x'_i

In plain English.

1. If two nodes have the same information set then they must be “simultaneous” decisions.
2. If these nodes are for the same players there must not be information generated in the past that makes them observationally different.

Example 2 *Violations of Perfect Recall.*

1. Which exit?

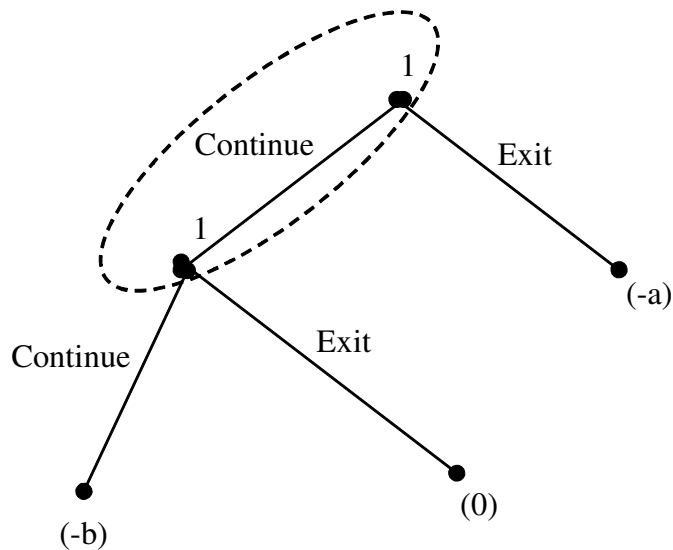
Violates (1) if don't remember whether you have passed an exit.

Leave bar (drunk—WHY ARE YOU DRIVING) then:

First exit—hotel

Second exit—home

No exit—Police take you to Jail



Obviously no pure strategy is optimal. Exit gives a sure utility of $-a$. Continue gives a sure utility of $-b$. Of the two exit is clearly better, but if you continue with a very small probability then you'll be almost sure

to exit when you want to go home. Of course there is an optimal mixed strategy:

$$\max_p -ap + p(1-p)0 - b(1-p)^2$$

Checking the first and second order conditions:

$$\begin{aligned} -a + 2b(1-p) &= 0 \\ -2b &< 0 \end{aligned}$$

The utility maximizing strategy is:

$$p = 1 - \frac{a}{2b}$$

The key thing is to recognize is that is strictly interior, neither pure strategy is optimal. But how is this person going to commit to this strategy? They must be self aware enough to recognize the problem. In the original framework they would just say "Oh, an exit, must be the first one."

2. Pie versus Cobbler

Violates (2), can't remember your own action.

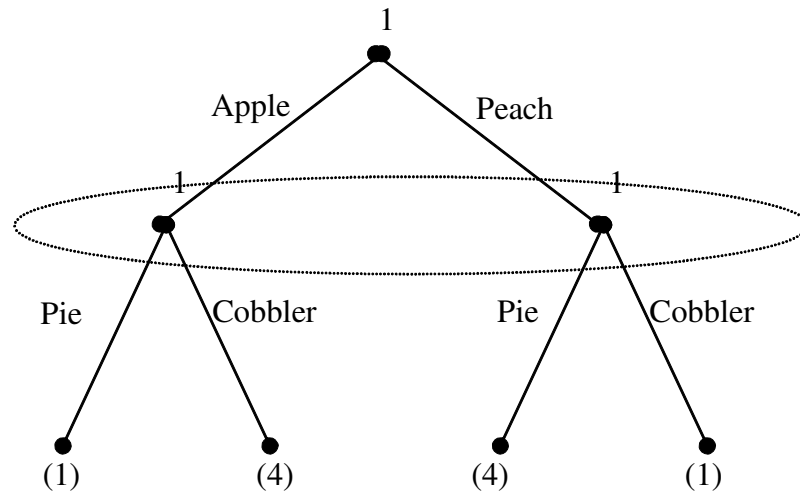
$$u(\text{Apple Pie}) > u(\text{Apple Cobbler})$$

$$u(\text{Apple Cobbler}) > u(\text{Apple Pie})$$

Week ago: Mother in Law asks Apple or Peach desert?

Now she has made Pie and Cobbler and you have forgotten what you chose. (And you can't tell by looking at them.)

The actions "Apple" and "Peach" lead to the same information set.



Now again, clearly, there is an optimal mixed strategy. Here there's no problem in carrying it out because you are aware she asked, just not what your answer was. Personally I'd just ask my mother in law what I said, she knows I'm a space cadet.

Notice that counter-examples are all decision problems, there is nothing about a *game* in the definition of perfect recall. Further notice there is an "optimal" strategy in both settings, but that the definition and concept of a strategy is a problem.