Econ 222-01
2013-2014 Spring
Homework 0
Due Date: You do not have to turn in this homework but you should finish
it before February 21st.

1) Consider a population consisting of the following four values: 1, 2, 3, and
4. Let $\bar{X}$ denote the sample mean of random samples of size 2 selected, with
replacement, from this population.

　　a. Write all the possible samples and their means.

　　b. Find the probability of all the sample means.

　　c. Plot the probability density function of $\bar{X}$.

　　d. Find the expected value $\bar{X}$.

　　e. Find the standard deviation of $\bar{X}$.

2) The random variable $X$ is normally distributed with a mean of 1 and variance
of 4. Find $P\left(0 \leq X \leq 1.5\right)$?

3) Time spent using e-mail per session is approximately normally distributed
with a mean of 8 minutes and standard deviation of 2 minutes. Let the
random variable $\bar{X}$ give the mean time spent using e-mail of random samples
of 25 sessions.

　　a. Find the expected value of $\bar{X}$, i.e., $\mu_{\bar{X}}$.

　　b. Find the standard deviation of $\bar{X}$, i.e., $\sigma_{\bar{X}}$.

　　c. What proportion of the sample means would be between 7.5 and 8.2
　　minutes?

4) Consider a normally distributed population with standard deviation 8. The
mean of the population is know to be less than or equal to 4 but the exact
value is not know. If we take a random sample of size 25 from this population
what is the largest value of the probability that the sample mean takes a value
as extreme as 6 (i.e., takes the value 6 or larger)?

5) We pick two numbers at random from a population which is uniformly
distributed over the interval $[0, 1]$. We would like to obtain the distribution
of the average of these two random numbers (variables). In order to get an
idea about this distribution, we use a spreadsheet program (LibreOffice,

EXCEL, etc.) to take $5,000$ pairs of random numbers from the interval $[0, 1]$ (using the `=rand()` function) and calculate their averages (thus we have $5,000$ observations of the random variable that we are interested with).

a. Construct a relative frequency distribution table using intervals of size $0.04$ (use the spreadsheet function `=countif` to construct this table).

b. Using the spreadsheet chart functions construct the frequency polygon corresponding to the table you obtain above.

c. The above is a approximation (a result obtained by simulation) of the density function of the random variable $(X_1 + X_2)/2$ where $X_1$ and $X_2$ are two independent random variables which are uniformly distributed over the interval $[0, 1]$. What do you think the exact density function of this random variable is?