Surviving Einstein

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Introduction

- A big debate is raging on in the philosophy of space and time
- The parties are the substantivalists (neo-Newtonians) and relationalists (neo-Leibnizians).
- Kant's view is not a live option.
- There is a sense of historical injustice.

Two goals

1. To review Kant's arguments about space.
2. To clarify whether Kant's theory of space is relevant today

Outline

Leibniz' relationalism

- Space is not an object.
- Assertions about space are meaningful, but they are but assertions about relations between co-existing material objects.
- Consequence: there is no empty space (and also no vacuum).
- (Consequence: there is no absolute motion.)

Kant *vs.* Leibniz

Incongruence

1. The left hand and the right hand share all their intrinsic geometrical properties.
2. But they cannot be made to coincide by a rigid motion.
3. Therefore, incongruence cannot be explained relationally.

Kant *vs.* Leibniz

The nature of geometry

1. If space is the order of co-existing material bodies, then geometry is empirical.
2. But geometry is not empirical.
3. Therefore, relationalism is false.

That is, Leibniz can explain the application of geometry, but only at the price of making it another empirical science.

Substantivalism

- Space is an immutable, necessary, eternal object.
- It is a *sensorium* of God.
- Space has geometrical structure independent of the material objects it contains.
- Consequence: there is empty space.
- (Consequence: there is absolute motion.)

Kant *vs.* Newton

- Some commentators doubt that there are any arguments against Newton.
- Objections against relationalism are consistent with Newton's views.
- Even incongruence can be explained if we posit absolute space and absolute directions, such as 'west', 'east', 'south' and 'north'.
- Perhaps Newton's views are not philosophically serious.

Kant *vs.* Newton

We can read two arguments into the *Prolegomena* (with little textual evidence).

Applicability of geometry

1. If geometry describes the nature of absolute space, then it is *a priori*.
2. But how can we apply it to the physical world?

That is, the apriority of geometry is rightly not in doubt, but we cannot explain the fact of its usefulness in physics.

Transcendental realism

If geometry describes the nature of absolute space, then we cannot explain our (certain) knowledge of it.

Master Argument

1. The representation of objects as distinct from me or distinct one from another presupposes the representation of space.
2. Experience presupposes the representation of objects as as distinct from me or distinct one from another.
3. Therefore, the representation of space is not given in experience—it is *a priori*.

Space as pure intuition

- Geometrical judgements are not analytical.
- Hence, space is not given in concepts.
- Hence, space must be given in intuition.
- And according to the Master Argument, it must be given in *a priori* intuition.

Corollary

The applicability of geometry is explained, so that our account is better than Newton's.And at the same time the apriority of geometry is preserved, so that our account is better than Leibniz'.

Non-Euclidean geometries

- Kant's assumption is that the space featuring in all these arguments is by necessity Euclidean.
- That is not an assumption that he could question. No alternative geometries were known at the time.
- But now we can distinguish between pure and physical geometry.
- Pure Euclidean geometry is an axiomatic system.Its axioms in themselves will be neither true nor false. Its theorems will be derived by means of logical rules from the axioms.
- It has a model: manifolds with zero curvature.
- But whether our world is such is a contingent issue.

SPACE or Euclidean space?

Should we interpret Kant's theory as a theory of generic SPACE or a theory of Euclidean space?

If it is a theory of SPACE. . .

- The fate of the Master Argument is an open question.
- But the applicability of geometry becomes trivial.Certainly, *some* geometry must apply in our study of nature.

SPACE or Euclidean space?

If the theory is about the Euclidean space. . .

- The Master Argument does not get off the ground.
- The claim about the applicability of geometry is translated as a claim about the physical applicability of Euclidean geometry.
- The general relativity theory shows not only there is no apodictic certainty in Euclidean geometry *qua* physical geometry—
- There are models (universes) having non-Euclidean and satisfying Einstein's equations.
- Euclidean universes (flat manifolds) give merely one possible solution of the equations.
- This rebuts a quasi-Kantian conventionalist response of Poincaré.

Where the general relativity is mute

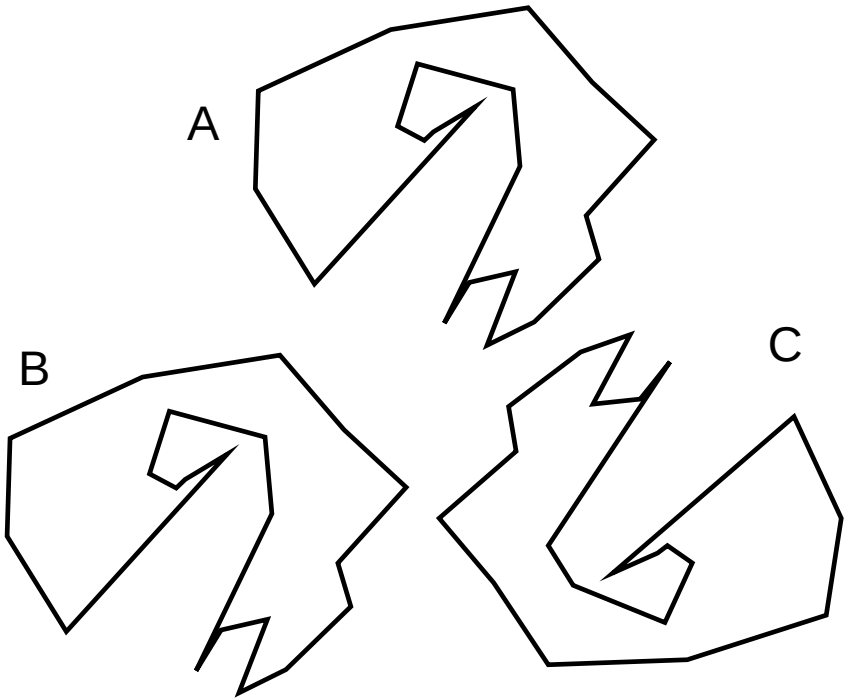
- Equations of general relativity are differential equations. They are sensitive to the local geometry of the universe, but not to its global topology.
- Essentially, only two local topological constraints are imposed: homogeneity and isotropy.
- This leaves possibility for many alternative global topologies.
- That is, the overall shape of space is not determined by general relativity.

Incongruent counterparts

Kant: There is a genuine intrinsic right-left difference which is graspable intuitively.

But what kind of apodictic pure intuition is in play here?

For instance, I may also intuit that a large complicated figure *A* is incongruent with another complicated figure *B*, whereas in fact they *are* congruent.



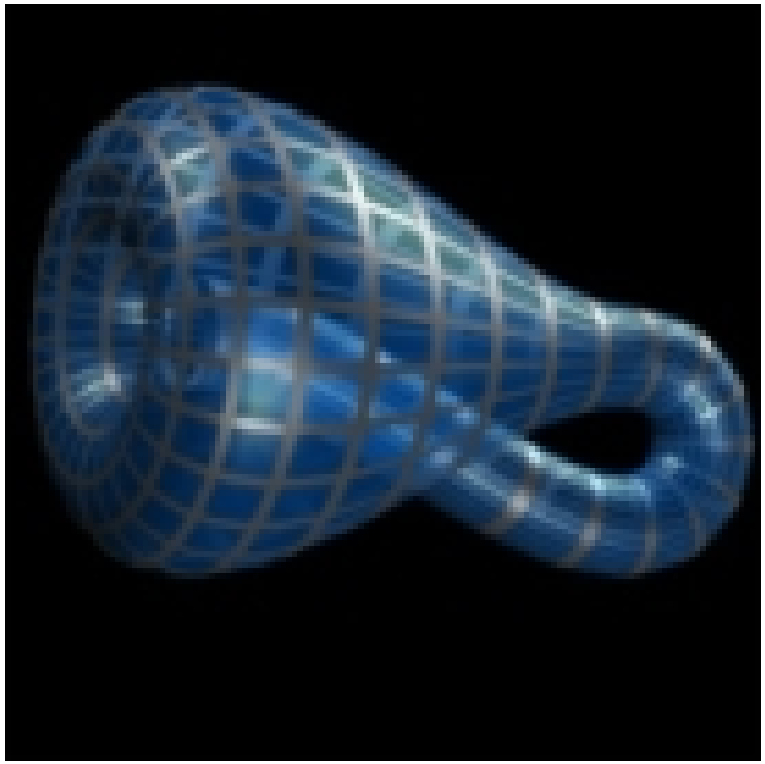
Incongruent counterparts: rotation

- There is no explanation why my intuition in the case of right-left figures gets it right. So, something else is involved in the right-left difference.
- Rotation in a four-dimensional space may transform right hands into left hands.
- Very much like the flat shape $\sim$ can be transformed into $\backsim$ by rotating it in the third dimension.
- Objection: If a 2-figure is rotated in 3-space, it must be embedded (how?) in a 3-space.Would it be the same figure as if it were embedded in 2-space? (Consider triangles on the plane and on the sphere.)
- Reply: Yes, the same, if the geometries of 2-space and 3-space are the same.

Incongruent counterparts: orientability

- The right-left incongruence depends on the global topological properties of space.
- If our space is non-orientable (if it contains Möbius strip), then there is no intrinsic left-right difference.

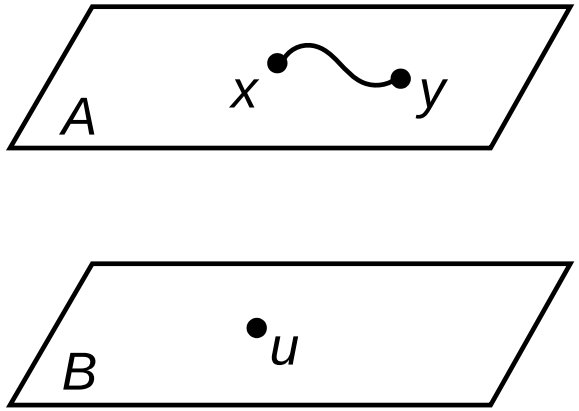
Incongruent counterparts: orientability

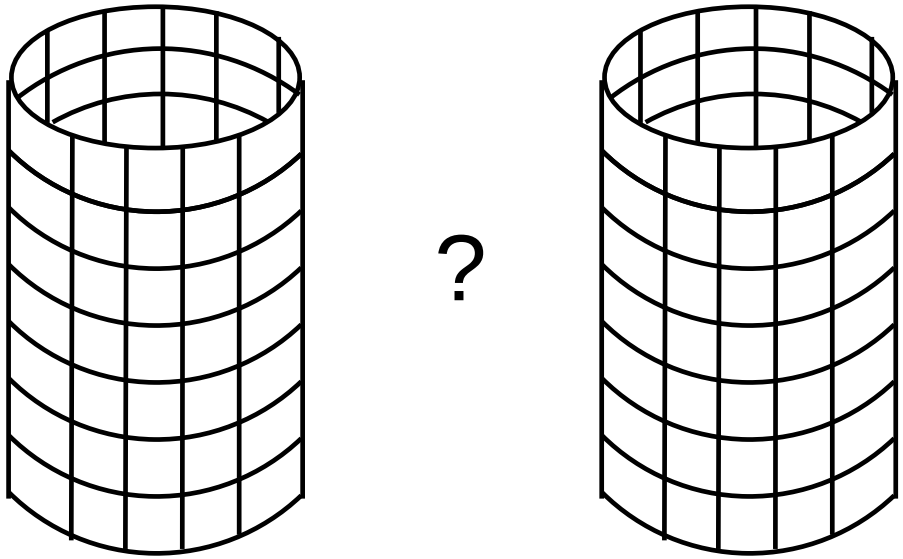


The unity of space

- Kant: We cannot represent multiple spaces, although we can represent multiple parts of the same space.
- The possibility of multiple spaces is entertained in modern physical theory.
- But does it make sense?
- The arguments, however, are physical.

The unity of space





Conclusion

- Not much is left from Kant's geometrical arguments themselves.
- However, a totally different question is whether there are certain non-analytic *a priori* principles at the foundation of physical theories.
- Kant was good at identifying the problematic points in the discussion, but not quite successful at resolving them.