

Weyl and Reichenbach on apriority

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Motivation

Friedman's programme

Reichenbach on apriority

Holism with continuity

Weyl on apriority

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- ▶ What is the status of the statements such as 'Phosphorus melts at 44°C ' or 'Light has the same velocity in all inertial frames'?
 - ▶ Mere arbitrary conventions.
 - ▶ Empirical discoveries.
 - ▶ *A priori* truths.
- ▶ What is the scope of Kripke's metaphysical necessity, and what is its relation to nomic necessity?
- ▶ The fate of Kuhn's incommensurability.

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[Friedman, 2001] presents a sustained attempt of reviving the rationalist tradition. *Some* laws of nature are known *a priori*.

The tripartite distinction

Level I The concepts and principles of natural science which face the 'tribunal of experience' via a rigorous process of empirical testing.

Level II The constitutively *a priori* principles that define the fundamental spatiotemporal framework within which the formulation and testing of empirical principles is possible.

Level III Philosophical meta-paradigms.

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Friedman, Quine, Kuhn

Friedman resists both Quinean holism and Kuhn's doctrine of incommensurability.

Contra Quine: mathematical beliefs are *a priori* and properly physical are empirical. There is a third class of beliefs characterised as relative *a priori* which mediates between mathematical and physical parts of the theory.

Contra Kuhn: historical developments in mathematical physics show that there is far more continuity in successive paradigms than Kuhn allows. The fact of continuity is to be explained by the class of relative *a priori* beliefs.

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Reichenbach on apriority

Reichenbach finds two senses of *a priori* in Kant.

Apriority qua necessity

S is *a priori* true iff *S* is true at all times.

Apriority qua coordination

S is *a priori* true iff the truth-value of *S* is justified through constitutive rules of experience. (Thus, *S* is a 'coordinating principle'.)

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Apriority *qua* coordination

- ▶ Upon fixing the coordinate cross we can determine a one-parameter family of curves by the algebraic function $f(x, y, z) = 0$.
- ▶ The Euclidean metric which fixes the relation that is to be obtained for a collection of spatial points to form a spatially extended body.
- ▶ We coordinate certain mathematical symbols for vectors with physical forces and thereby conceive the latter as objects having vector-like properties.

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To say that coordinating principles (e.g. Euclidean axioms) are not necessarily true is to say that they are *revisable*. But revisability does not amount to 'total' falsehood. We should replace coordinating principles with those principles which differ from them only with respect to a limited instances of the available data:

All laws contain coordinating principles, and if we wish to test new coordinating principles inductively, we must first change every physical law. It would indeed be nonsensical to test new principles by means of experiences still presupposing the old principles. Such a procedure would be technically impossible. We cannot start physics all over again. [Reichenbach, 1965, 67-8]

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Cumulative development

The Folk View

Newer theories account for more and more phenomena and get closer and closer to truth.

- ▶ Kepler's laws are approximations for Newton's laws; the laws of stationary electric and magnetic fields are approximations for Maxwell's equations.
- ▶ The theory of relativity and quantum mechanics contains Newtonian mechanics as an approximate limiting instance for low velocities and sufficiently massive bodies moving in smoothly varying fields.

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Beyond the Folk View

- ▶ Reichenbach's view is close to the Folk View; the reasons he gives are purely pragmatic.
- ▶ Reichenbach cannot accommodate Kuhn's data on incommensurability.
- ▶ Kuhn's data is most effective in the case of those theories that did not have a strong—or any—mathematical apparatus at their disposal.
- ▶ As soon as mathematical concepts and mathematical formalism make their entry, Kuhn's picture becomes less appealing.

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Continuity by sharing form

$$x' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{ux}{c^2}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$x' = x - ut$$

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$$x' = x \cos \theta + y \sin \theta$$

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The concept of force

Coulomb's Law

$$\mathbf{F}_1 = \frac{1}{4\pi} \frac{q_1 q_2}{r_{12}^2} \mathbf{r}_{12}.$$

- ▶ If it is supposed to explain the concept of force, it involves action at a distance.
- ▶ This is unintelligible.

[We] bow to the dictates of the theory of knowledge. Even Leibniz formulated the principle of continuity, of infinitely near action, as a general principle, and could not, for this reason, become reconciled to Newton's Law of Gravitation, which entails action at a distance and which corresponds fully to that of Coulomb. [Weyl, 1922, 66]

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Field equations

$$\mathbf{F} = e \cdot \mathbf{E}.$$
$$\mathbf{E} = \sum_j \frac{1}{4\pi} \frac{q_j}{r_{1j}^2} \mathbf{r}_{1j}.$$

$$\mathbf{E} = \frac{1}{4\pi} \int \frac{\rho \cdot \mathbf{r}}{r^3} dV.$$

$$\phi = -\frac{1}{4\pi} \int \frac{\rho}{r} dV,$$

$$\mathbf{E} = -\nabla\phi = -\text{grad } \phi.$$

Field equations

The concept of force is explained by the field equations:

$$\nabla \times \mathbf{E} = \text{curl } \mathbf{E} = 0.$$

$$\nabla \cdot \mathbf{E} = \text{div } \mathbf{E} = \rho.$$

Here a knowledge of the function in an arbitrarily small region surrounding the point-charge suffices to determine the differential quotient of the function at that point.

- ▶ Therefore, we have infinitely near action congruent with the law of continuity.
- ▶ Coulomb's Law is a merely mathematical consequence of the field equations.

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Blank forms and holism

The fundamental law of mechanics is a blank form which acquires a concrete content only when the conception of force occurring in it is filled in by physics. [Weyl, 1922, 66-7]

The laws constitute a cycle. We require this whole network of theoretical considerations to arrive at an experimental means of verification. [Weyl, 1922, 67]

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Weyl on symbolic constructions

- ▶ Science progresses towards increased symbolisations, both to increase generality (assist predictions) and to exclude a subjective element.
- ▶ Symbolic notation is systematic. A mathematical activity starts when one can provide a construction of indefinitely reproducing the given operation (think of Hilbert's strokes).
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References

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