Bilkent University
Department of Mathematics

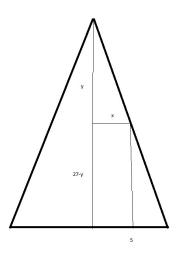Instructor: Ali Sinan Sertöz   YOUR NAME:

In this quiz you can use only pencils and erasers.
Show your work in detail, unless only an answer is required. Correct answer without proper explanation does not receive any partial credits.

Q-1) A right circular cylinder is inscribed in a cone of height 27cm and base radius 5cm. Find the largest possible volume of such a cylinder.

Answer:

Using the above cross-section and similar triangles we get

$$\frac{y}{x} = \frac{27}{5}, \quad \text{hence} \quad y = \frac{27}{5}\, x.$$

Volume of this cylinder is $\pi x^2 (27 - y)$. Substituting the value of y in terms of x we get

$$V(x) = 27\pi x^2 \left(1 - \frac{x}{5}\right), \quad 0 \leq x \leq 5.$$

We want to maximize this function, so we calculate its derivative and equate to zero.

$$V'(x) = 27\pi x \left(2 - \frac{3}{5}x\right) = 0 \;\Rightarrow\; x = \frac{10}{3}.$$

We check that $V(0) = V(5) = 0$ at the end points, so this root must give the maximal value since $V(x)$ is always non-negative on the interval. Hence the maximal volume is

$$V(\frac{10}{3}) = 100\pi.$$