Quiz # 7
Math 101-Section **011** Calculus I
24 November 2016, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

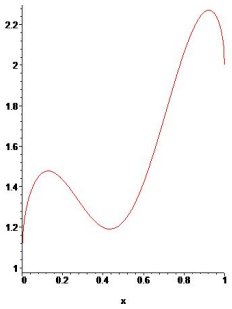
Bilkent University

Your Name: .....

Student ID: ..... Your Department: .....

Show your work in detail. Correct answers without justification are never graded.

Q-1) Let $f(x) = x\sqrt{1-x^2} + \cos^2(\pi x) + x^{1/3}$. Evaluate the area under the graph of $y = f(x)$ and above the x-axis on the interval $[0, 1]$. Here is the graph of $y = f(x)$ on the interval $[0, 1]$. (10 points)



Answer: Substituting $u = 1 - x^2$ we get $du = -2x\,dx$ and

$$\int_0^1 x\sqrt{1-x^2}\,dx = \frac{1}{2}\int_0^1 u^{1/2}\,du = \left(\frac{1}{3}u^{3/2}\Big|_0^1\right) = \frac{1}{3}.$$

Using the half angle formula we have

$$\cos^2(\pi x) = \frac{1+\cos(2\pi x)}{2}.$$

We then have

$$\int_0^1 \cos^2(\pi x)\,dx = \int_0^1 \left(\frac{1}{2} + \frac{\cos(2\pi x)}{2}\right)\,dx = \left(\frac{x}{2} + \frac{\sin(2\pi x)}{4\pi}\Big|_0^1\right) = \frac{1}{2}.$$

Finally

$$\int_0^1 x^{1/3}\,dx = \left(\frac{3}{4}x^{4/3}\Big|_0^1\right) = \frac{3}{4}.$$

Now adding these we get the area.

$$\int_0^1 f(x)\,dx = \frac{1}{3} + \frac{1}{2} + \frac{3}{4} = \frac{19}{12}.$$