



Quiz # 9  
Math 101-Section 011 Calculus I  
8 December 2016, Thursday  
Instructor: Ali Sinan Sertöz  
**Solution Key**



Bilkent University

Your Name: .....

Student ID: .....

Your Department: .....

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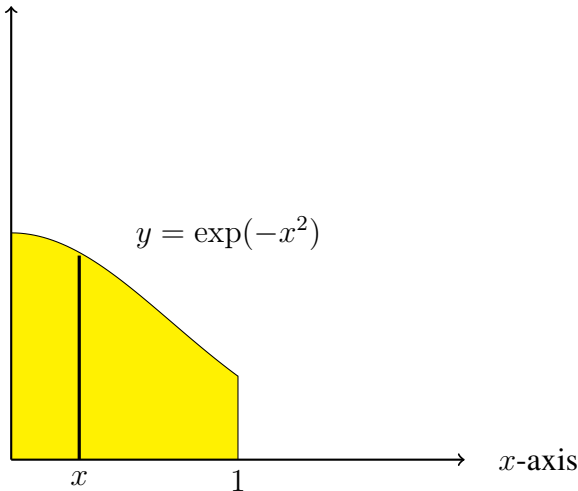
*Show your work in detail. Correct answers without justification are never graded.*

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**Q-1)** Let  $D$  be the region bounded by the curves  $y = e^{-x^2}$ ,  $y = 0$ ,  $x = 0$  and  $x = 1$ . Find the volume of the solid obtained by revolving the region  $D$  around the  $y$ -axis. (10 points)

**Answer:**

$y$ -axis

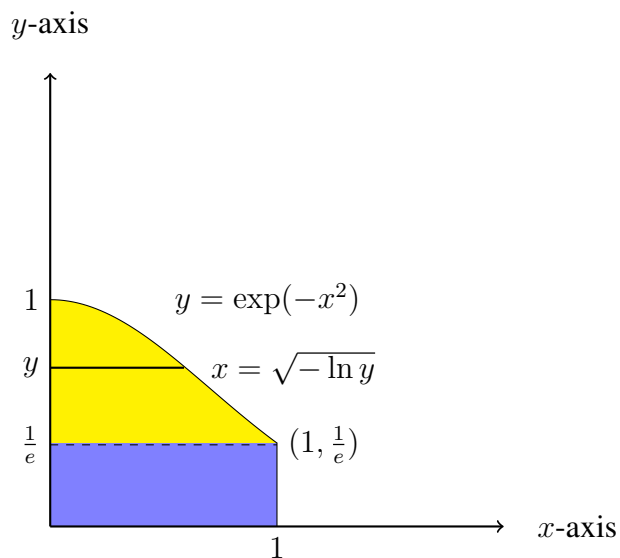


Using cylindrical shells method seems convenient for this problem. The height of the above thick line is  $e^{-x^2}$ , its thickness is  $dx$  and it travels a distance of  $2\pi x$  around the  $y$ -axis. Hence its incremental volume is  $2\pi x e^{-x^2}$ . We now *add* all these volumes between  $x = 0$  and  $x = 1$  to find the total volume.

$$V = 2\pi \int_0^1 x e^{-x^2} dx = \pi \left( -e^{-x^2} \Big|_0^1 \right) = \pi(1 - e^{-1}) \approx 1.985865304.$$

In the second step we used the substitution  $u = -x^2$  to find an anti-derivative of  $x e^{-x^2}$

For another solution see next page.



The volume obtained by revolving the above colored region around the  $y$ -axis is the sum of the volumes obtained by revolving the blue and yellow regions separately.

The blue region revolved around the  $y$ -axis gives a cylinder of volume  $\pi/e$ .

For the yellow region we note that the length of the thick line which is the radius of the circle of revolution is  $x = \sqrt{-\ln y}$ . Hence the volume of that solid of revolution is

$$\pi \int_{1/e}^1 \ln y \, dy = \left( y \ln y - y \Big|_{1/e}^1 \right) = \pi \left( 1 - \frac{2}{e} \right).$$

Adding these two volumes we find

$$V = \frac{\pi}{e} + \pi \left( 1 - \frac{2}{e} \right) = \pi \left( 1 - \frac{1}{e} \right) \approx 1.985865304.$$