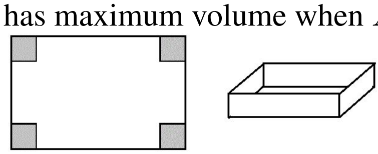
Quiz # 7
Math 101-Section **01** Calculus I
30 March, 2018, Friday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Name: ....

Department: .... Student ID: ....

Q-1) We want to construct an open top box by cutting off squares from the corners of a rectangular sheet of paper and then folding, as in the figure.
Assume that the dimensions of the sheet are $0 < A < B$ units. Find the dimensions of the box that has maximum volume when $A = 8$ and $B = 15$ units.



Answer:

Let the side of the square to be cut off be x. Then the volume is

$$V(x) = x(A-2x)(B-2x) = 4x^3 - 2(A+B)x^2 + ABx, \ 0 \le x \le A/2.$$

And

$$V'(x) = 12x^2 - 4(A+B)x + AB = 0 \text{ when } x_1 = \frac{1}{6}(A+B-\sqrt{A^2-AB+B^2}), \ x_2 = \frac{1}{6}(A+B+\sqrt{A^2-AB+B^2}).$$

It is easy to check that

$$0 < x_1 < A/2 < x_2.$$

Since $V(0) = 0$ and $V(A/2) = 0$, this critical point gives the absolute maximum value. In fact you can also check that $V''(x) > 0$ on the given interval.

Finally, the maximum volume is obtained when $x = x_1$, and then

$$V(x_1) = \frac{1}{18}(A^2B + AB^2) - \frac{1}{27}(A^3 + B^3) + \frac{1}{27}(A^2 - AB + B^2)^{3/2}.$$

Now putting in $A = 8$, $B = 15$ gives $x_1 = 5/3$.

To get full marks, you have to:

1. Correctly formulate the problem with the domain
2. Determine the critical points inside the domain
3. Argue correctly about where the maximum volume occurs.