Quiz # 8
Math 101-Section **01** Calculus I
6 April, 2018, Friday
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Solution Key

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Q-1)

(i) Express $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^2+kn+n^2}{k^3+2n^3}$ as a definite integral; do not evaluate the integral

(ii) Evaluate $\int_0^{\pi/2}(\sin x-\sin^3 x)(7+\cos^3 x)^{1/3}\,dx$.

Answer:

(i) We want the summand to look like $f(x_k)\Delta x_k$ where $x_k = k/n$ and $\Delta x_k = 1/n$.

$$\frac{k^2+kn+n^2}{k^3+2n^3} = \frac{k^2n+kn^2+n^3}{k^3+2n^3}\frac{1}{n} = \frac{\left(\frac{k}{n}\right)^2+\frac{k}{n}+1}{\left(\frac{k}{n}\right)^3+2}\frac{1}{n} = \frac{x^2+x+1}{x^3+2}\,\Delta x.$$

Hence

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^2+kn+n^2}{k^3+2n^3} = \int_0^1\frac{x^2+x+1}{x^3+2}\,dx.$$

(ii) Let $u = 7+\cos^3 x$, then $du = -3\sin x\cos^2 x\,dx$

$$\begin{aligned}
\int_{x=0}^{x=\pi/2}(\sin x-\sin^3 x)(7+\cos^3 x)^{1/3}\,dx &= \int_{x=0}^{x=\pi/2}\sin x\cos^2 x\,(7+\cos^3 x)^{1/3}\,dx\\
&= -\frac{1}{3}\int_{x=0}^{x=\pi/2}(-3\sin x\cos^2 x)(7+\cos^3 x)^{1/3}\,dx\\
&= -\frac{1}{3}\int_{x=0}^{x=\pi/2}u^{1/3}\,du\\
&= -\frac{1}{3}\left(\frac{3}{4}u^{4/3}\Big|_{x=0}^{x=\pi/2}\right)\\
&= -\frac{1}{3}\left(\frac{3}{4}(7+\cos^3 x)^{4/3}\Big|_{x=0}^{x=\pi/2}\right)\\
&= 4-\frac{7\,7^{1/3}}{4}\\
&\approx 0.65.
\end{aligned}$$