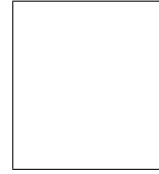




Quiz # 3
Math 101-Section 09 Calculus I
16 October 2018, Friday
Instructor: Ali Sinan Sertöz
Solution Key



Bilkent University

Q-1) Angle θ depends on time as $\theta(t) = \frac{\pi}{15} t$, where t is in seconds and all distances are in centimeters

We have two points in the plane given as $P(t) = (\sin \theta, \cos \theta)$ and $Q = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. How fast is the point P moving away from the point Q when $t = 5$?

Solution:

Let $d(t)$ be the distance between P and Q . Then

$$d(t)^2 = (\sin \theta + \frac{1}{\sqrt{2}})^2 + (\cos \theta - \frac{1}{\sqrt{2}})^2 = 2 + \sqrt{2}(\sin \theta - \cos \theta).$$

Taking derivatives of both sides with respect to t , we get

$$2d(t)d'(t) = \sqrt{2}(\cos \theta + \sin \theta)\theta'(t) = \sqrt{2}(\cos \theta + \sin \theta)\frac{\pi}{15}.$$

We find that

$$d(5)^2 = 2 + \sqrt{2}\left(\frac{\sqrt{3}-1}{2}\right),$$

and

$$2d(5)d'(5) = \sqrt{2}\left(\frac{\sqrt{3}+1}{2}\right)\frac{\pi}{15}.$$

Hence

$$d'(5) = \frac{\sqrt{2}(\sqrt{3}+1)\pi}{15\sqrt{8+2\sqrt{2}(\sqrt{3}-1)}} \approx 0.25.$$

Thus the point P is moving away from point Q with a speed of 0.25 centimeters per second.