Quiz # 7
Math 101-Section **09** Calculus I
23 November 2018, Friday
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Solution Key

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Q-1) A huge spherical snowball of radius $6\ m$ begins to melt. It melts at a rate proportional to its surface area. After $30\ min$ its radius becomes $1\ m$. Find much longer will it take to melt completely.

Solution:

Let $R(t)$ denote the radius of the snowball at time t, $S(t)$ the surface area and $V(t)$ the volume. We have

$$V(t) = \frac{4\pi}{3}R(t)^3, \quad S(t) = 4\pi R(t)^2.$$

That the snowball melts at a rate proportional to surface area means that there is a constant α such that

$$V'(t) = \alpha S(t).$$

This gives

$$4\pi R(t)^2 R'(t) = 4\alpha\pi R(t)^2, \quad \text{or} \quad R'(t) = \alpha.$$

Thus

$$R(t) = \alpha t + C, \quad \text{for some constant } C.$$

But we know that $R(0) = 6$, so we have

$$R(t) = \alpha t + 6, \quad \text{where } t \text{ is in minutes.}$$

We are given that $R(30) = 1$. This forces $\alpha = -1/6$ and we finally have

$$R(t) = -\frac{t}{6} + 6.$$

Next we want to find t such that $R(t) = 0$. This gives $t = 36$. Since it already took $30\ min$ for the radius to become $1\ m$, it will take $6\ min$ more for the snowball to melt completely.