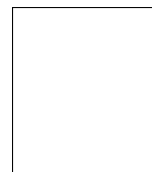




Quiz # 7  
Math 101-Section 13 Calculus I  
22 November 2018, Thursday  
Instructor: Ali Sinan Sertöz  
**Solution Key**



Bilkent University

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**Q-1)** A huge spherical snowball of radius  $9\text{ m}$  begins to melt. It melts at a rate proportional to its surface area. After  $5\text{ h}$  its surface area becomes  $36\pi\text{ m}^2$ . Find much longer will it take to melt completely.

**Solution:**

Let  $R(t)$  denote the radius of the snowball at time  $t$ ,  $S(t)$  the surface area and  $V(t)$  the volume. We have

$$V(t) = \frac{4\pi}{3}R(t)^3, \quad S(t) = 4\pi R(t)^2.$$

That the snowball melts at a rate proportional to surface area means that there is a constant  $\alpha$  such that

$$V'(t) = \alpha S(t).$$

This gives

$$4\pi R(t)^2 R'(t) = 4\alpha\pi R(t)^2, \quad \text{or } R'(t) = \alpha.$$

Thus

$$R(t) = \alpha t + C, \quad \text{for some constant } C.$$

But we know that  $R(0) = 9$ , so we have

$$R(t) = \alpha t + 9, \quad \text{where } t \text{ is in hours.}$$

We are given that  $S(5) = 36\pi$ . This forces  $\alpha = -6/5$  and we finally have

$$R(t) = -\frac{6}{5}t + 9.$$

Next we want to find  $t$  such that  $R(t) = 0$ . This gives  $t = 15/2 = 7.5$ . Since it already took  $5\text{ h}$  for the surface area to become  $36\pi\text{ m}^2$ , it will take  $2\text{ h}$  and  $30\text{ min}$  more for the snowball to melt completely.