Quiz # 7
Math 101-Section **13** Calculus I
22 November 2018, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Q-1) A huge spherical snowball of radius $9\ m$ begins to melt. It melts at a rate proportional to its surface area. After $5\ h$ its surface area becomes $36\pi\ m^2$. Find much longer will it take to melt completely.

Solution:

Let $R(t)$ denote the radius of the snowball at time t, $S(t)$ the surface area and $V(t)$ the volume. We have

$$V(t) = \frac{4\pi}{3}R(t)^3, \quad S(t) = 4\pi R(t)^2.$$

That the snowball melts at a rate proportional to surface area means that there is a constant α such that

$$V'(t) = \alpha S(t).$$

This gives

$$4\pi R(t)^2 R'(t) = 4\alpha\pi R(t)^2, \quad \text{or} \quad R'(t) = \alpha.$$

Thus

$$R(t) = \alpha t + C, \quad \text{for some constant } C.$$

But we know that $R(0) = 9$, so we have

$$R(t) = \alpha t + 9, \quad \text{where } t \text{ is in hours.}$$

We are given that $S(5) = 36\pi$. This forces $\alpha = -6/5$ and we finally have

$$R(t) = -\frac{6}{5}t + 9.$$

Next we want to find t such that $R(t) = 0$. This gives $t = 15/2 = 7.5$. Since it already took $5\ h$ for the surface area to become $36\pi\ m^2$, it will take $2\ h$ and $30\ min$ more for the snowball to melt completely.