Quiz # 8
Math 101-Section **13** Calculus I
29 November 2018, Thursday
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Solution Key

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Q-1) Evaluate $\displaystyle\int_0^1 x\sqrt{1-\sqrt{x}}\,dx$.

Solution: Let $u = 1-\sqrt{x}$. Then $x = (1-u)^2$ and $dx = -2(1-u)du$. When x goes from 0 to 1, u goes from 1 to 0. We then have

$$\begin{aligned}
\int_0^1 x\sqrt{1-\sqrt{x}}\,dx &= 2\int_0^1 (1-u)^3 u^{1/2}\,du \\
&= 2\int_0^1 (1-3u+3u^2-u^3)u^{1/2}\,du \\
&= \int_0^1 (u^{1/2}-3u^{3/2}+3u^{5/2}-u^{7/2})\,du \\
&= 2\left(\frac{2}{3}u^{3/2}-\frac{6}{5}u^{5/2}+\frac{6}{7}u^{7/2}-\frac{2}{9}u^{9/2}\Big|_0^1\right) \\
&= \frac{64}{315} \approx 0.203.
\end{aligned}$$