Quiz # 10
Math 101-Section **09** Calculus I
14 December 2018, Friday
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Solution Key

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Q-1) For any positive integer n, let $A(n)$ denote the area bounded by the curves $y = \dfrac{\ln x}{x}$ and $y = \dfrac{(\ln x)^{2n}}{x}$. Find $\displaystyle\lim_{n \to \infty} A(n)$.

Solution:
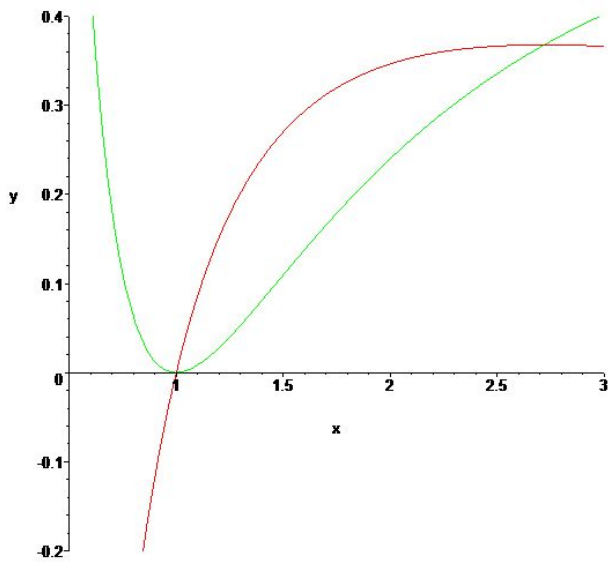
The two curves intersect at $x = 1$ and $x = e$, as can be easily solved. Hence

$$
\begin{aligned}
A(n) &= \int_1^e \left[\frac{\ln x}{x} - \frac{(\ln x)^{2n}}{x}\right] dx \\
&= \frac{1}{2}\left((\ln x)^2\Big|_1^e\right) - \frac{1}{2n+1}\left((\ln x)^{2n+1}\Big|_1^e\right) \\
&= \frac{1}{2} - \frac{1}{2n+1}.
\end{aligned}
$$

Now it is clear that

$$
\lim_{n \to \infty} A(n) = \frac{1}{2}.
$$