Bilkent University

Quiz # 06
Math 101-Section 08 Calculus I
7 November 2019, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Consider the function $f(x) = x^3 + 2x^2 + x + 1$ on $[-2, 1]$.

(i) Find the critical points of f. (2 pts)

(ii) Find the inflection points of f. (1 pt)

(iii) Find the local min/max points of f. (2 pts)

(iv) Find global min/max values of f. (2 pts)

(v) Find regions where the graph of $y = f(x)$ is concave up/down. (2 pts)

(vi) Plot $y = f(x)$. (1 pt)

Solution:

(i) $f'(x) = 3x^2 + 4x + 1 = 0$ gives $x = -1/3$ and $x = -1$ as critical points.

(ii) $f''(x) = 6x + 4 = 0$ gives $x = -2/3$. At this point $f''(x)$ changes sign, so this is an inflection point.

(iii) $f(-1) = 1$, $f''(-1) = -2$, so $x = -1$ is a local max point.
$f(-1/3) = 22/27$, $f''(-1/3) = 2$, so $x = -1/3$ is a local min point.

(iv) $f(-2) = -1$, $f(-1) = 1$, $f(-1/3) = 22/27$, $f(1) = 5$. Hence the max value of f is 5 , and the min value is -1.

(v) $f''(x) < 0$ on $[-2, -2/3)$, so here the graph is concave down, and $f''(x) > 0$ on $(-2/3, 1]$, so here the graph is concave up.

(vi)

Here is a graph of $y = f(x)$.