Bilkent University

Quiz # 07
Math 101-Section 08 Calculus I
14 November 2019, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) The line $y = 3x + 4$ intersects the parabola $y = x^2$ at the points A and B. What is the maximum area a triangle ΔABC can have, where C is a point on the parabola below the given line?

Hint: The area of a triangle with vertices $A = (a_1, a_2)$, $B = (b_1, b_2)$ and $C = (c_1, c_2)$ is one half the absolute value of $(c_1 - a_1)(b_2 - a_2) - (b_1 - a_1)(c_2 - a_2)$.

Solution:

We solve $x^2 = 3x + 4$ to find the x-coordinates of the intersection. This gives $A = (-1, 1)$ and $B = (4, 16)$. Let $C = (t, t^2)$ be a point on the parabola below the given line. This means $-1 \leq t \leq 4$.

Using the hint, we want to find the extreme points of

$$f(t) = -5t^2 + 15t + 20, \text{ for } t \in [-1, 4]$$

We find that $f'(t) = -10t + 15 = 0$ when $t = 3/2$.

Evaluating f at this critical point and the end points gives

$$f(-1) = 0, \quad f(3/2) = 125/4, \quad f(4) = 0.$$

Hence the maximal possible area is half of $125/4$ which is $125/8 = 15.625$.

Here is a graph of $y = f(t)$, $-1 \leq t \leq 4$.