



Bilkent University

Quiz # 01  
Math 101-Section 12 Calculus I  
27 September 2020 Sunday  
Instructor: Ali Sinan Sertöz  
**Solution Key**

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**Q-1)** Let  $f(x) = 27x^3 - 108x^2 + 117x - 28$ . Use the Intermediate Value Theorem to show that there exist at least two solutions to  $f(x) = 0$  when  $0 \leq x \leq 2$

**Solution:** We evaluate  $f$  on some easy points of  $[0, 2]$ .

$$f(0) = -28, f(1) = 8, f(2) = -10.$$

By the Intermediate Value Theorem (IVT), there is a root between  $x = 0$  and  $x = 1$ . Similarly there must be another root between  $x = 1$  and  $x = 2$ .

It is not required in this quiz but if we put  $x = 3$  into  $f$ , we find  $f(3) = 80$ . Since there is another sign change here, there must exist a root between  $x = 2$  and  $x = 3$ .

Since this is a third degree polynomial, it does not have more than three real roots, so these are all the roots of  $f(x) = 0$ .

In fact, the roots are  $x = 1/3 \approx 0.33$ ,  $x = 4/3 \approx 1.33$  and  $x = 7/3 \approx 2.33$ .

Here is a graph of  $y = f(x)$  for your information, not required as part of this quiz.

