Bilkent University

Quiz # 07
Math 101-Section 12 Calculus I
29 November 2020 Sunday
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Solution Key

Q-1) Calculate $\lim_{n\to\infty} \sum_{k=0}^{n} \frac{n+k}{n\sqrt{3n^2+8nk+4k^2}}$.

Solution:

We interpret this as the limit of a Riemann sum of some function $f(x)$.

Since we have $n+k$, we first think of converting this to $1+\frac{k}{n}$. This then suggest that the integral starts from $x=1$ when $k=0$, to $x=2$ when $k=n$.

Now we try to write the given expression in terms of $1+\frac{k}{n}$. But first we take out $\frac{1}{n}$ since it is going to be the common length of each subinterval.

We thus have

$$\frac{n+k}{n\sqrt{3n^2+8nk+4k^2}} = \frac{1}{n}\,\frac{(1+\frac{k}{n})}{\sqrt{4(1+\frac{k}{n})^2-1}} = \frac{1}{n} f((1+\frac{k}{n})),$$

where

$$f(x) = \frac{x}{\sqrt{4x^2-1}}.$$

Thus we have

$$\lim_{n\to\infty} \sum_{k=0}^{n} \frac{n+k}{n\sqrt{3n^2+8nk+4k^2}} = \int_1^2 \frac{x}{\sqrt{4x^2-1}}\,dx = \left(\frac{1}{4}\sqrt{4x^2-1}\Big|_1^2 \right) = \frac{1}{4}\left(\sqrt{15}-\sqrt{3}\right) \approx 0.5352\ldots$$