Bilkent University

Quiz # 08
Math 101-Section 12 Calculus I
6 December 2020 Sunday
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Solution Key

Q-1) Calculate the area enclosed by the curves $f(x) = x^3 - 5x^2 - x + 5$ and $g(x) = x^3 - 4x^2 - 5x$ on the interval $[-4, 8]$.

Solution:

We first notice that

$$f(x) = x^3 - 5x^2 - x + 5 = (x-1)(x+1)(x-5),$$
$$g(x) = x^3 - 4x^2 - 5x = x(x+1)(x-5).$$

Therefore these curves intersect at $x = -1$ and $x = 5$.

We check that:
On $[-4, -1)$, we have $g(x) - f(x) > 0$,
on $(-1, 5)$, we have $f(x) - g(x) > 0$, and
on $(5, 8]$, we have $g(x) - f(x) > 0$.

Therfore the area can be calculated as follows.

$$\begin{aligned}
Area &= \int_{-4}^{-1} (g(x) - f(x))\, dx + \int_{-1}^{5} (f(x) - g(x))\, dx + \int_{5}^{8} (g(x) - f(x))\, dx \\
&= \int_{-4}^{-1} (x^2 - 4x - 5)\, dx + \int_{-1}^{5} (-x^2 + 4x + 5)\, dx + \int_{5}^{8} (x^2 - 4x - 5)\, dx \\
&= \left(\frac{1}{3}x^3 - 2x^2 - 5x \Big|_{-4}^{-1} \right) + \left(-\frac{1}{3}x^3 + 2x^2 + 5x \Big|_{-1}^{5} \right) + \left(\frac{1}{3}x^3 - 2x^2 - 5x \Big|_{5}^{8} \right) \\
&= 36 + 36 + 36 \\
&= 108.
\end{aligned}$$

Here is a sketch of the graphs of $y = f(x)$ and $y = g(x)$, not reqired as part of this quiz.