Bilkent University

Quiz # 03
Math 101-Section 12 Calculus I
21 October 2021 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Consider the hyperbola $x^2 - y^2 = 9$. Observe that the point $p_0 = (5, -4)$ is on this hyperbola.

(i) Find y' at $(5, -4)$.

(ii) Find y'' at $(5, -4)$.

(iii) Write an equation of the form $ax + by = c$ for the tangent line at p_0 for this hyperplane.

(iv) Find the area of the triangle bounded by the above tangent line, the x-axis and the y-axis.

Show your work. Simplify as much as possible.

Grading: (i) 3 points, (ii) 3 points, (iii) 1 point, (iv) 3 points.

Solutions:

(i) From $x^2 - y^2 = 9$, by implicitly differentiating with respect to x we find $2x - 2yy' = 0$. This gives $y' = \dfrac{x}{y}$. Hence at p_0 we get $y' = -\dfrac{5}{4}$.

(ii) This time from $2x - 2yy' = 0$ again by implicit differentiation with respect to x we get $2 - 2y'y' - 2yy'' = 0$. Putting in $(x, y, y') = (5, -4, -\dfrac{5}{4})$ we get $y'' = \dfrac{9}{64}$.

(iii) An equation of this tangent line is $y + 4 = -\dfrac{5}{4}(x - 5)$ which simplifies to $5x + 4y = 9$.

(iv) From $5x + 4y = 9$ we find that the x and y intercepts are $(\dfrac{9}{5}, 0)$ and $(0, \dfrac{9}{4})$. Hence the area is $A = \dfrac{1}{2}\dfrac{9}{4}\dfrac{9}{5} = \dfrac{81}{40}$.