Bilkent University

Quiz # 04
Math 101-Section 12 Calculus I
4 November 2021 Thursday
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Solution Key

Q-1) Let $f(x) = 2x^3 - 3x^2 - 12x + 1$ where $-2 \leq x \leq 1$. Find the minimum and maximum values of f. The Mean Value Theorem guarantees that there is a point $a \in (-2, 1)$ such that the slope of the tangent line to the curve $y = f(x)$ at $x = a$ is equal to the slope of the line joining the points $(-2, f(-2))$ and $(1, f(1))$. Find such an a.
Show your work. Simplify as much as possible.

Solutions:

We start by taking the derivative of f.

$$f'(x) = 6x^2 - 6x - 12 = 6(x+1)(x-2) = 0 \text{ when } x = -1 \text{ or } x = 2.$$

We observe that only $x = -1$ is in our domain. We then evaluate f at the end points and at the only critical point in the domain.

$$f(-2) = -3, \quad f(-1) = 8, \quad f(1) = -12.$$

By looking at these values we see that the minimum value of f is -12, and the maximum value is 8 on this interval.

The slope of the line joining the points $(-2, f(-2)) = (-2, -3)$ and $(1, f(1)) = (1, -12)$ is

$$\frac{(-3) - (-12)}{(-2) - (1)} = -3.$$

We are looking for a point a such that $f'(a) = -3$ and $-2 < a < 1$.

Solving for $f'(x) = -3$ we find two solutions

$$a = \frac{1}{2}(1 - \sqrt{7}) \approx -0.82 \text{ and } b = \frac{1}{2}(1 + \sqrt{7}) \approx 1.82.$$

The one which is in our interval is a. Here is a relevant graph: