



Bilkent University

Quiz # 09  
Math 101-Section 12 Calculus I  
16 December 2021 Thursday  
Instructor: Ali Sinan Sertöz  
**Solution Key**

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**Q-1)** Find the values of  $a$ ,  $b$  and  $c$  such that

$$\lim_{x \rightarrow 0} \left( \frac{\sin x}{x^5} + a + \frac{b}{x^2} + \frac{c}{x^4} \right) = 0.$$

*Show your work.*

**Solutions:** In the following solution  $\stackrel{LH}{=}$  means that we are applying L'Hospital's rule at that stage.

$$\begin{aligned} \lim_{x \rightarrow 0} \left( \frac{\sin x}{x^5} + a + \frac{b}{x^2} + \frac{c}{x^4} \right) &= \lim_{x \rightarrow 0} \left( \frac{\sin x + cx + bx^3 + ax^5}{x^5} \right) = \left[ \frac{0}{0} \right] \\ &\stackrel{LH}{=} \lim_{x \rightarrow 0} \left( \frac{\cos x + c + 3bx^2 + 5ax^4}{5x^4} \right) = \left[ \frac{0}{0} \right] \quad \text{when } c = -1 \\ &\stackrel{LH}{=} \lim_{x \rightarrow 0} \left( \frac{-\sin x + 6bx + 20ax^3}{20x^3} \right) = \left[ \frac{0}{0} \right] \\ &\stackrel{LH}{=} \lim_{x \rightarrow 0} \left( \frac{-\cos x + 6b + 60ax^2}{60x^2} \right) = \left[ \frac{0}{0} \right] \quad \text{when } b = \frac{1}{6} \\ &\stackrel{LH}{=} \lim_{x \rightarrow 0} \left( \frac{\sin x + 120ax}{120x} \right) = \left[ \frac{0}{0} \right] \\ &\stackrel{LH}{=} \lim_{x \rightarrow 0} \left( \frac{\cos x + 120a}{120} \right) = 0 \quad \text{when } a = -\frac{1}{120}. \end{aligned}$$