



Bilkent University

Quiz # 06
Math 101-Section 12 Calculus I
17 November 2022 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1)

(i) Evaluate the integral $\int_0^1 x^{22} dx$ using the Fundamental Theorem of Calculus.

(ii) Write a Riemann sum for this integral.

(iii) Calculate the limit of this Riemann sum.

Hint: $1^m + 2^m + \dots + n^m = \frac{n^{m+1}}{m+1} + \frac{n^m}{2} + \text{lower degree terms in } n$

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 3+3+4=10 points.

Solution:

(i) $\int_0^1 x^{22} dx = \left(\frac{x^{23}}{23} \Big|_0^1 \right) = \frac{1}{23}.$

(ii) $\int_0^1 x^{22} dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left(\frac{i}{n} \right)^{22}.$

(iii)

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left(\frac{i}{n} \right)^{22} = \lim_{n \rightarrow \infty} \frac{1}{n^{23}} \sum_{i=1}^n i^{22} = \lim_{n \rightarrow \infty} \frac{1}{n^{23}} \left(\frac{n^{23}}{23} + \frac{n^{22}}{2} + \text{lower degree terms in } n \right) = \frac{1}{23}.$$