Bilkent University

Quiz # 07
Math 101-Section 08 Calculus I
25 November 2022 Friday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Let P be the parabola $y = x^2$, L_1 be the line $3y = 2x + 16$, and L_2 be the line $y = 8 - 2x$.
Let A be the area bounded by P and L_1, B be the area bounded by P and L_2, and C be the area that lies above P but below both L_1 and L_2.
Evaluate only the integral in (iii).

(i) Find the points of intersection of P with L_1 and L_2 as well as the point of intersection of L_1 with L_2

(ii) Write a definite integral which calculates A.

(iii) Write a definite integral which calculates B.

(iv) Write a definite integral which calculates C.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 5+2+1+2=10 points.

Solution:

(i) $P \cap L_1 = \{(-2, 4), (8/3, 64/9)\}$, $P \cap L_2 = \{(-4, 16), (2, 4)\}$, $L_1 \cap L_2 = \{(1, 6)\}$.

(ii)

$$A = \int_{-2}^{8/3} [(\frac{2}{3}x + \frac{16}{3}) - (x^2)]\, dx.$$

(iii)

$$B = \int_{-4}^{2} [(8 - 2x) - (x^2)]\, dx = \left(8x - x^2 - \frac{x^3}{3} \Big|_{-4}^{2} \right) = 36.$$

(iv)

$$C = \int_{-2}^{1} [(\frac{2}{3}x + \frac{16}{3}) - (x^2)]\, dx + \int_{1}^{2} [(8 - 2x) - (x^2)]\, dx.$$