



Bilkent University

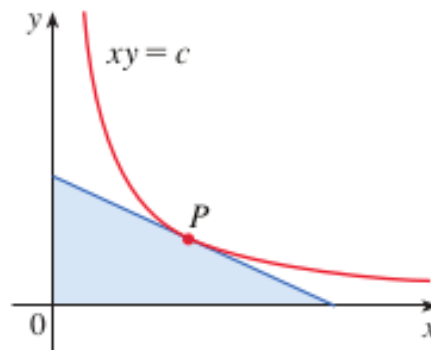
Quiz # 02
Math 101-Section 05 Calculus I
5 October 2023 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1)

Exercise 116 on page 147 of your textbook: A tangent line is drawn to the hyperbola $xy = c$, where c is a positive constant, at an arbitrary point P as shown in the figure.

(a) Show that the midpoint of the line segment cut from this tangent line by the coordinate axes is P .

(b) Show that the triangle formed by the tangent line and the coordinate axes, the shaded region, always has the same area, no matter where P is located on the hyperbola.



Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 5+5=10 points if satisfactory explanations are provided.

Solution:

(a) Let the coordinates of the point P be $(t, c/t)$ for some arbitrary non-zero constant t . The slope of the hyperbola $y = c/x$ at any point is $y' = -c/x^2$, hence at P the slope is $-c/t^2$. The equation of the tangent line at P is then $y = (-c/t^2)(x - t) + c/t$. This tangent line intersects the x -axis at $(2t, 0)$, and the y -axis at $(0, 2c/t)$. The midpoint of the line joining these two intercepts is then easily seen to be P .

(b) The area of the shaded region is $(1/2)(2t)(2c/t) = 2c$, which is clearly independent of the t coordinate of P .