Quiz # 05
Math 101-Section 04 Calculus I
26 October 2023 Thursday
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Solution Key

Bilkent University

Q-1) Let $h(x) = 7 - \dfrac{20}{x^2 + 4}$ on the interval $[-4, 4]$.

    (a) Find the absolute minimum and the absolute maximum values of h on the given interval.

    (b) Let f be a function such that $f'(x) = h(x)$ on the given interval. Show that

$$16 \le f(4) - f(-4) \le 48.$$

Grading: 5+5=10 points

Solution:

(a) $h'(x) = \dfrac{40x}{(x^2 + 4)^2} = 0$ gives $x = 0$ as the only critical point.

We evaluate h at the critical and end points.

$$h(-4) = 6, \quad h(0) = 2, \quad h(4) = 6.$$

Hence the absolute minimum value of h is 2 at $x = 0$, and the absolute maximum value of h is 6 at $x = \pm 4$.

(b) Using the Mean Value Theorem for f on the interval $[-4, 4]$ we get

$$\frac{f(4) - f(-4)}{4 - (-4)} = f'(c), \quad \text{for some } c \in (-4, 4).$$

But $f'(c) = h(c)$ and $2 \le h(c) \le 6$. Thus we get

$$2 \le \frac{f(4) - f(-4)}{4 - (-4)} \le 6,$$

which simplifies to

$$16 \le f(2) - f(-1) \le 48,$$

as claimed.