Bilkent University

Quiz # 05
Math 101-Section 05 Calculus I
26 October 2023 Thursday
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Solution Key

Q-1) Let $h(x) = 5 - \dfrac{2}{x^2+1}$ on the interval $[-1, 2]$.

(a) Find the absolute minimum and the absolute maximum values of h on the given interval.

(b) Let f be a function such that $f'(x) = h(x)$ on the given interval. Show that

$$9 \leq f(2) - f(-1) \leq \frac{69}{5}.$$

Grading: 5+5=10 points

Solution:

(a) $h'(x) = \dfrac{4x}{(x^2+1)^2} = 0$ gives $x = 0$ as the only critical point.

We evaluate h at the critical and end points.

$$h(-1) = 4, \quad h(0) = 3, \quad h(2) = \frac{23}{5}.$$

Hence the absolute minimum value of h is 3 at $x = 0$, and the absolute maximum value of h is $\dfrac{23}{5}$ at $x = 2$.

(b) Using the Mean Value Theorem for f on the interval $[-1, 2]$ we get

$$\frac{f(2) - f(-1)}{2 - (-1)} = f'(c), \quad \text{for some } c \in (-1, 2).$$

But $f'(c) = h(c)$ and $3 \leq h(c) \leq \dfrac{23}{5}$. Thus we get

$$3 \leq \frac{f(2) - f(-1)}{2 - (-1)} \leq \frac{23}{5},$$

which simplifies to

$$9 \leq f(2) - f(-1) \leq \frac{69}{5},$$

as claimed.