



Bilkent University

Quiz # 08  
Math 101-Section 04 Calculus I  
16 November 2023 Thursday  
Instructor: Ali Sinan Sertöz  
**Solution Key**

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**Q-1)** Find a function  $f$  and a number  $a$  such that

$$2023 + \int_a^x \frac{f(t)}{1 + \sin^2 t^4} dt = 7\sqrt{x} \quad \text{for all } x > 0.$$

Hint: Use the Fundamental Theorem of Calculus Part 1. (This question is inspired by exercise 81 on page 338 of your book.) Grading: 10 points

**Solution:** (Grader: taha.yigit@ug.bilkent.edu.tr)

Take the derivative of both sides with respect to  $x$  to obtain

$$\frac{f(x)}{1 + \sin^2 x^4} = \frac{7}{2\sqrt{x}},$$

which gives

$$f(x) = \frac{7(1 + \sin^2 x^4)}{2\sqrt{x}}.$$

Next go to the given equation again and now put  $x = a$  on both sides to obtain

$$2023 = 7\sqrt{a},$$

which gives

$$a = 289^2 = 83521.$$