Bilkent University
Quiz \# 09
Math 101-Section 04 Calculus I
23 November 2023 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Consider the curves $y=x^{2}$ and $y=6-x$.
(a) Set up the integral which gives the area of the region between these curves on $[-3,6]$.
(b) Set up the integral which gives the volume of the solid obtained by revolving the region of part (a) around the $x$-axis.
(c) Set up the integral which gives the volume of the solid obtained by revolving around the $y$-axis the region between these curves on $[-3,0]$.
(d) Evaluate one of the above integrals.

Grading: $3+3+3+1=10$ points
Solution: (Grader: rburakguler71@gmail.com)
Here is the graph:

(a) : Area $=\int_{-3}^{2}\left[(6-x)-\left(x^{2}\right)\right] d x+\int_{2}^{6}\left[\left(x^{2}\right)-(6-x)\right] d x=\frac{125}{6}+\frac{184}{3}=\frac{493}{6}$.
(b) : Volume $=\pi \int_{-3}^{2}\left[(6-x)^{2}-\left(x^{2}\right)^{2}\right] d x+\pi \int_{2}^{6}\left[\left(x^{2}\right)^{2}-(6-x)^{2}\right] d x=\frac{500}{3} \pi+\frac{22912}{15} \pi=\frac{25412}{15} \pi$.
(c) : Volume $=\pi \int_{0}^{6}(\sqrt{y})^{2} d y+\pi \int_{6}^{9}\left[(\sqrt{y})^{2}-(6-y)^{2}\right] d y=18 \pi+\frac{27}{2} \pi=\frac{63}{2} \pi$.
$\left(\mathbf{c}^{\prime}\right):$ Volume $=2 \pi \int_{-3}^{0}(-x)\left[(6-x)-\left(x^{2}\right)\right] d x=\frac{63}{2} \pi$.

