Math 102 Homework-2
Due Date: 23 July 2008 Wednesday
Either hand in your homework solutions in class or put them in my mail box until 17:00 on Wednesday.

Q-1) Show that the vector field

$$\mathbf{F} = \big(-\tan(x+y^2+z^3),\ -2y\tan(x+y^2+z^3),\ -3z^2\tan(x+y^2+z^3)\big)$$

is conservative. Find a potential function for $\mathbf{F}$ and evaluate the integral

$$\int_{(0,0,0)}^{(1,2,3)} \mathbf{F}\cdot\mathbf{T}\,d\sigma.$$

Q-2) let $\mathbf{F} = \left(\dfrac{2x}{x^2+y^4}, \dfrac{4y^3}{x^2+y^4}\right)$. Evaluate $\int_C \mathbf{F}\cdot\mathbf{T}\,ds$, where C is the circle of radius R centered at the origin. Beware here that the Green's theorem does not hold since $\mathbf{F}$ is not defined at the origin. Observe that in this problem $M_y = N_x$ for the vector field $\mathbf{F} = (\mathbf{M}, \mathbf{N})$. Suppose you have the task of providing such vector fields on demand. How would you construct such vector fields without much effort? How did I *invent* the above vector field?

Q-3) Find the area of the surface S cut from the cone $z^2 = 4x^2+4y^2$, $z \geq 0$, by the cylinder $x^2+y^2 = 2x$.

Q-4) Evaluate the integral

$$\int\int_S \nabla\times\mathbf{F}\cdot\mathbf{n}\,d\sigma$$

where S is the level surface given by $x^2+z^2-4(x+z)-y+8=0$, $0 \leq y \leq 4$, and

$$\mathbf{F} = \left(x^2z+\ln(y^2+1),\ \cosh(x^2+y^2)-\ln(z^2+1),\ \frac{y^3}{y^2+1}-xz^2\right).$$

Q-5) Solve the very last problem of the book, exercise 21 on page 1228.

Please send comments and questions to `sertoz@bilkent.edu.tr`