



Quiz # 10
Math 102-011 Calculus
Bring to class on 5 May 2015, Tuesday



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YOUR NAME:

In this quiz you can use fingers, calculators or smart phones to do your calculations.

However show your work in detail. Correct answer without proper explanation does not receive any partial credits.

Q-1) Let D be a region in the plane, and let $f(x, y) = xy$. It is given that

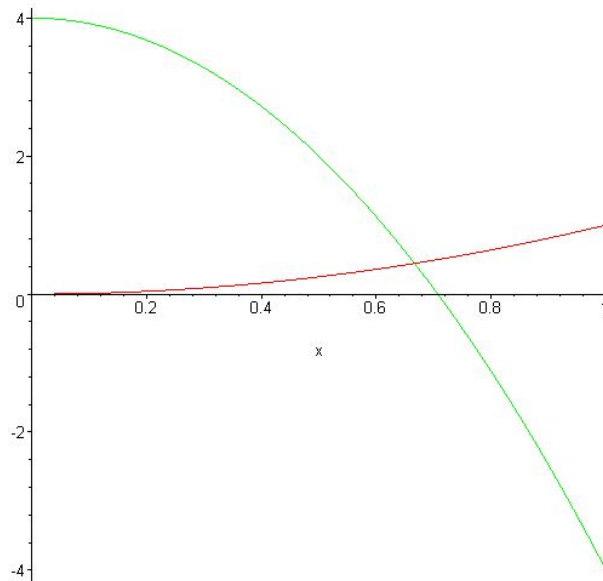
$$\iint_D f = \int_0^{4/9} \int_0^{\sqrt{y}} xy \, dx \, dy + \int_{4/9}^4 \int_0^{\sqrt{(4-y)/8}} xy \, dx \, dy.$$

- (i) Sketch the region D .
- (ii) Rewrite the above double integral in the order $dydx$.
- (iii) Evaluate the double integral $\iint_D f$.

: Grading is 30+30+40 points.

Answer:

(i) The region D is bounded by the curves $y = x^2$, $y = 4 - 8x^2$ and $x = 0$.



The two parabolas intersect at the point $(2/3, 4/9)$.

(ii) Reversing the order of integration we get

$$\iint_D f = \int_0^{2/3} \int_{x^2}^{4-8x^2} xy \, dy \, dx.$$

(iii) Integrating the above integral we find

$$\begin{aligned} \iint_D f &= \int_0^{2/3} \int_{x^2}^{4-8x^2} xy \, dy \, dx \\ &= \int_0^{2/3} \left(\frac{1}{2} x^2 y \Big|_{x^2}^{4-8x^2} \right) dx \\ &= \int_0^{2/3} \left(\frac{63}{2} x^5 - 32 x^3 + 8 x \right) dx \\ &= \left(\frac{21}{4} x^6 - 8 x^4 + 4 x^2 \Big|_0^{2/3} \right) \\ &= \frac{160}{243} \approx 0.658. \end{aligned}$$