Quiz # 1
Math 102-011 Calculus
February 13, 2015 Friday

Bilkent University

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Q-1)

(i) Show that $\lim_{n\to\infty} \frac{\ln n}{n^{1/4}} = 0$. *[30 points]*

(ii) Show that $\ln n < n^{1/4}$ for all large n. *[30 points]*

(iii) Does the series $\sum_{n=2}^{\infty} \left(\frac{\ln n}{n}\right)^2$ converge or diverge? *[40 points]*

Show your work in detail. Only correct solutions will be graded; correct answers without justification are never graded.

Answer:

Let $f(x) = \frac{\ln x}{x^{1/4}}$. Then using L'Hopital's rule we have

$$\lim_{n\to\infty} f(n) = \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{4}{x^{1/4}} = 0.$$

This means that for all large n we have $\frac{\ln n}{n^{1/4}} < 1$, which answers the second part. Using this we have for all large n,

$$\left(\frac{\ln n}{n}\right)^2 < \left(\frac{n^{1/4}}{n}\right)^2 < \frac{n^{1/2}}{n^2} = \frac{1}{n^{3/2}}.$$

The series $\sum_{n=2}^{\infty} \frac{1}{n^p}$ converges for $p = 3/2 > 1$ by p-test and dominates our series for large n, so our series converges by comparison test.