



Quiz # 11  
 Math 102-Section 06 Calculus II  
 11 May 2017, Thursday  
 Instructor: Ali Sinan Sertöz  
**Solution Key**



Bilkent University

Your Name: .....

Student ID: .....

Your Department: .....

*Show your work in detail. Correct answers without justification are never graded.*

**Q-1)** Let  $f(x) = e^{x^3}$ . Find  $f^{(n)}(0)$  for all  $n = 0, 1, 2, \dots$

**Answer:** Recall that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

where the series converges to  $e^x$  for all  $x$ . We can therefore replace  $x$  by  $x^3$  and still have equality for all  $x$ .

$$e^{x^3} = 1 + x^3 + \frac{x^6}{2!} + \frac{x^9}{3!} + \dots + \frac{x^{3n}}{n!} + \dots$$

Compare this series with the Taylor series of  $f(x)$ .

$$e^{x^3} = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k.$$

Comparing the two series term by term we discover that

$$f^{(n)}(0) = \begin{cases} \frac{n!}{k!} & \text{if } n = 3k, \\ 0 & \text{if } 3 \nmid n. \end{cases}$$