Bilkent University

Quiz # 04
Math 102-Section **10** Calculus II
7 March 2019, Thursday
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Solution Key

Q-1) Let $f(x,y)=x^4-xy^2+y^3$. This function has two critical points, $(0,0)$ and $p_0=(x_0,y_0)\neq(0,0)$.

(i) Find p_0.

(ii) Determine if p_0 is a local minimum, local maximum or a saddle point.

(iii) Does f have a global maximum?

(iv) Does f have a global minimum?

Grading: (i) 2 points, (ii) 4 points, (iii) 2 points, (iv) 2 points.

Solution:

(i)

$$f_x=4x^3-y^2=0,\quad f_y=-2xy+3y^2=0 \quad\Rightarrow\quad p_0=(1/9,2/27).$$

(ii)

$$\begin{aligned}
&f_{xx}=12x^2,\quad f_{yy}=-2x+6y,\quad f_{xy}=-2y,\\
&\Delta(x,y)=f_{xx}f_{yy}-f_{xy}^2=-24\,x^3+72\,x^2y-4\,y^2,\\
&\Delta(1/9,2/27)=\frac{8}{729}>0 \quad\text{and}\quad f_{xx}(1/9,2/27)=\frac{4}{27}>0.
\end{aligned}$$

Hence by the Second Derivative Test, p_0 is a local minimum point.

(iii) $\lim\limits_{x\to\infty} f(x,0)=\infty$, so the function has no global maximum.

(iv) $\lim\limits_{y\to-\infty} f(0,y)=-\infty$, so the function has no global minimum.