



Bilkent University

Quiz # 07
Math 102-Section 10 Calculus II
4 April 2019, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1)

- (i) Draw or describe the graph given in spherical coordinates as $\rho = \cos \phi$.
- (ii) Draw or describe the graph given in spherical coordinates as $\phi = \pi/6$.
- (iii) Write a triple integral in spherical coordinates which calculates the volume of the solid that lies inside $\rho = \cos \phi$ but above $\phi = \pi/6$.
- (iv) Evaluate this integral.

Grading: (i) 2 points, (ii) 2 points, (iii) 4 points, (iv) 2 points.

Solution:

- (i) This is the sphere of radius $1/2$ with center at $(0, 0, 1/2)$ in \mathbb{R}^3 .
- (ii) This is the half cone whose sides make an angle of $\pi/6$ with the z -coordinate and lie above the xy -plane.
- (iii)

$$V = \int_0^{2\pi} \int_0^{\pi/6} \int_0^{\cos \phi} \rho^2 \sin \phi \, d\rho d\phi d\theta.$$

(iv)

$$\begin{aligned} V &= \int_0^{2\pi} \int_0^{\pi/6} \int_0^{\cos \phi} \rho^2 \sin \phi \, d\rho d\phi d\theta \\ &= \int_0^{2\pi} \int_0^{\pi/6} \sin \phi \left(\frac{1}{3} \rho^3 \Big|_0^{\cos \phi} \right) d\phi d\theta = \int_0^{2\pi} \int_0^{\pi/6} \frac{1}{3} \sin \phi \cos^3 \phi \, d\phi d\theta \\ &= \int_0^{2\pi} \left(-\frac{1}{12} \cos^4 \phi \Big|_0^{\pi/6} \right) d\theta \\ &= \int_0^{2\pi} \frac{7}{192} d\theta \\ &= \frac{7\pi}{96} \approx 0.23. \end{aligned}$$