Bilkent University

Quiz # 08
Math 102-Section **10** Calculus II
11 April 2019, Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Consider the solid that is common to the cylinders $x^2+y^2=a^2$, $\quad x^2+z^2=a^2$, $\quad z^2+y^2=a^2$, where $a>0$.

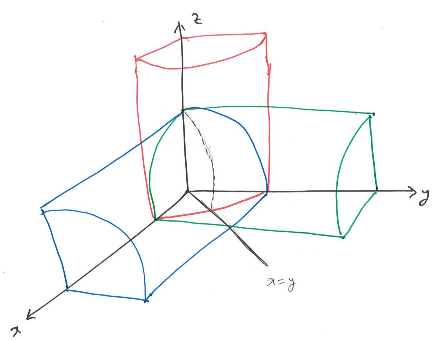
(i) Sketch the part of the solid as seen in the first octant.

(ii) Set up a triple integral which calculates the volume of this solid.

(iii) Evaluate this integral.

Grading: (i) 3 points, (ii) 5 points, (iii) 2 points.

Solution:



(i) $x^2+y^2=a^2$, $\quad y^2+z^2=a^2$, $\quad z^2+x^2=a^2$.

(ii) In the xy-plane consider the region that is bounded by $y=0$, $y=x$ and $x^2+y^2=a^2$. The solid that lies above this region is one sixteenth of the whole solid. A generic ray emanating from a point in this region and parallel to the z-axis leaves the solid along the (green) surface $x^2+z^2=a^2$. Notice that $x=y$ and $x^2+y^2=a^2$ curves intersect in the xy-plane at $x=y=a/\sqrt{2}$. We now set up the volume integral using this information.

$$V=16\left[\int_0^{a/\sqrt{2}}\int_0^{x}\int_0^{\sqrt{a^2-x^2}}dz\,dy\,dx+\int_{a/\sqrt{2}}^{a}\int_0^{\sqrt{a^2-x^2}}\int_0^{\sqrt{a^2-x^2}}dz\,dy,\,dx\right]$$

(iii) Continuing with the above integral we have:

$$\begin{aligned} V &= 16\left[\int_0^{a/\sqrt{2}}x\sqrt{a^2-x^2}\,dx+\int_{a/\sqrt{2}}^{a}(a^2-x^2)\,dx\right] \\ &= 16\left[\left(-\frac{1}{3}(a^2-x^2)^{3/2}\Big|_0^{a/\sqrt{2}}\right)+\left(a^2x-\frac{1}{3}x^3\Big|_{a/\sqrt{2}}^{a}\right)\right] \\ &= 16\left[1-\frac{\sqrt{2}}{2}\right]a^3\approx 4.68a^3. \end{aligned}$$