Bilkent University

Quiz # 01
Math 102 Section 03 Calculus II
13 February 2020
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Solution Key

Q-1) Find an equation of the plane through the line of intersection of the planes

$$x + 2y + 3z = 6, \quad \text{and} \quad 4x + 5y + 8z = 17,$$

and is perpendicular to the plane $x + 3y + z = 2020$.

Solution:

Let L be the line of intersection of the given two planes $x + 2y + 3z = 6$ and $4x + 5y + 8z = 17$.

First we find a point on L. This can be done by trial and error. Formally set $z = t$ and solve the resulting two linear equations in x and y to find

$$x = \frac{4-t}{3}, y = \frac{7-4t}{3}, z = t.$$

Putting, for example $t = 1$, we get $p_0 = (1, 1, 1)$ on L.

The normal vectors for the given two planes are $\vec{n}_1 = (1, 2, 3)$ and $\vec{n}_2 = (4, 5, 8)$. Each being perpendicular to the line of intersection, $\vec{n}_1 \times \vec{n}_2$ is parallel to the line of intersection.

The normal vector of the plane $x + 3y + z = 2020$ is $\vec{n}_3 = (1, 3, 1)$, and since the plane we consider is perpendicular to this plane, $\vec{n}_3$ is also parallel to our plane.

At this stage we have two vectors that are parallel to the plane we want to write. These vectors are $\vec{n}_1 \times \vec{n}_2$ and $\vec{n}_3$.

Hence a normal vector for our plane is $(\vec{n}_1 \times \vec{n}_2) \times \vec{n}_3$. We now calculate this

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 4 & 568 & \end{vmatrix} = (1, 4, -3), \qquad (\vec{n}_1 \times \vec{n}_2) \times \vec{n}_3 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 4 & -3 \\ 1 & 3 & 1 \end{vmatrix} = (13, -4, -1).$$

Finally, the equation of the plane we are looking for is

$$(13, -4, -1) \cdot (x, y, z) = (13, -4, -1) \cdot (1, 1, 1),$$

or simply

$$13x - 4y - z = 8.$$