Bilkent University

Quiz # 02
Math 102 - Calculus II - Section 03
17 February 2022 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1)

(a) Find all values of $p > 0$ for which the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges.

(b) Show that the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)}$ converges.

(c) Let $\displaystyle s = \sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)}$ and $\displaystyle s_m = \sum_{n=2}^{m} \frac{(-1)^n}{n(\ln n)}$.

If $4 < \ln 100 < 5$, find an upper bound for $|s - s_{99}|$.

Grading: 3+4+3 points

Solutions:

(a) We consider the decreasing and continuous function $f(x) = \dfrac{1}{x(\ln x)^p}$ and use the integral test. When $p \neq 1$,

$$\int_2^{\infty} \frac{1}{x(\ln x)^p}\, dx = \left(\frac{(\ln x)^{1-p}}{1-p} \Bigg|_2^{\infty} \right).$$

This is finite only when $p > 1$. On the other hand when $p = 1$

$$\int_2^{\infty} \frac{1}{x(\ln x)}\, dx = \left(\ln \ln x \Big|_2^{\infty} \right) = \infty.$$

Hence this series converges only for $p > 1$, by the integral test

(b) Let $f(x) = \dfrac{1}{x \ln x}$, for $x \geq 2$. We have

$$f'(x) = -\frac{\ln x + 1}{(x \ln x)^2} < 0,$$

so the general term $a_n = \dfrac{1}{n \ln n}$ is decreasing. Moreover we clearly have

$$\lim_{n \to \infty} a_n = 0.$$

Hence the given series converges by the Alternating Series Test.

(c) We know that when an alternating series converges by the Alternating Series Test then, also using $4 < \ln 100 < 5$, we get

$$|s - s_{99}| < a_{100} = \frac{1}{100 \ln 100} < \frac{1}{400} = 0.0025.$$