Bilkent University

Quiz # 10
Math 102 - Calculus II - Section 03
21 April 2022 Thursday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Evaluate

$$I = \int_{\frac{-7\sqrt{\pi}}{2}}^{0} \int_{-y/7}^{\sqrt{\pi}/2} \cos x^2 \, dx \, dy + \int_{0}^{\frac{13\sqrt{\pi}}{2}} \int_{y/13}^{\sqrt{\pi}/2} \cos x^2 \, dx \, dy$$

using the following guide:

(a) Sketch the region covered by the above integrals.

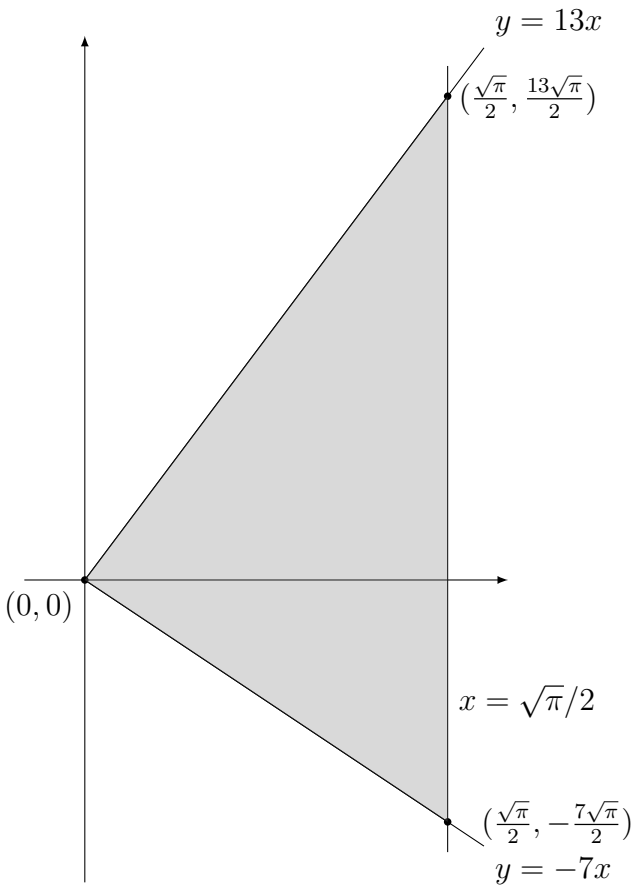
(b) Write the above I as a single integral with the reverse order of integration.

(c) Evaluate (easily!) the integral you found in the above step.

Grading: 4+3+3 points

Solutions:

(a) The region of integration is shaded in the figure below.



(b) When we reverse the order of integration we get

$$I = \int_0^{\sqrt{\pi}/2} \int_{-7x}^{13x} \cos x^2 \, dy \, dx.$$

(c) We can now evaluate the above integral.

$$\begin{aligned}
I &= \int_0^{\sqrt{\pi}/2} \int_{-7x}^{13x} \cos x^2 \, dy \, dx \\
&= \int_0^{\sqrt{\pi}/2} \left(y \cos x^2 \Big|_{y=-7x}^{y=13x} \right) \, dx \\
&= \int_0^{\sqrt{\pi}/2} (13x \cos x^2 + 7x \cos x^2) \, dx \\
&= \left(\frac{13}{2} \sin x^2 + \frac{7}{2} \sin x^2 \Big|_{x=0}^{x=\sqrt{\pi}/2} \right) \\
&= \frac{13}{2} \frac{1}{\sqrt{2}} + \frac{7}{2} \frac{1}{\sqrt{2}} \\
&= \frac{10}{\sqrt{2}} = 5\sqrt{2} \approx 7.071.
\end{aligned}$$