Bilkent University

Quiz # 3
Math 102-Section 11
31 March 2023, Friday, Moodle Quiz
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Solution Key

Q-1) Let $p_0 = (-1, 2, 3)$, $\vec{u} = (4, 5, 6)$ and $\vec{v} = (7, 8, 9)$. Let π be the name of the plane containing p_0 and not intersecting the lines parallel to $\vec{u}$ and $\vec{v}$. Write the equation of π in the format $x + By + Cz = D$, where B, C, D are numbers.

Q-2) Let $u = (1, 2, 3)$, $v = (4, 5, 6)$ and $w = (7, -8, -9)$. Let π be the name of the plane containing these points. Write the equation of π in the format $x + By + Cz = D$, where B, C, D are numbers.

Q-3) Let the parametric curve C be given as $p(t) = (t, t^2, t^3)$, $t \in \mathbb{R}$. This is called the twisted cubic in $\mathbb{R}^3$ since it cannot be embedded into $\mathbb{R}^2$ without losing its geometric properties. Let L be the tangent line to C at $t = 2$. Write vector, parametric and symmetric equations of L.

Show your work in detail. Correct answers without detailed explanation do not get any credit.

Grading: 3+3+4=10 points.

Solution:

(1) Since π is parallel to $\vec{u}$ and $\vec{v}$, the cross-product $\vec{u} \times \vec{v}$ gives us a normal direction

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = (-3, 6, -3) = -3(1, -2, 1),$$

so we can take $\vec{n} = (1, -2, 1)$. Then an equation for π is of the form

$$\vec{n} \cdot (x, y, z) = \vec{n} \cdot p_0,$$

which gives

$$x - 2y + z = -2.$$

(2) Since $p = v - u = (3, 3, 3)$ and $q = w - u = (6, -10, -12)$ are parallel to π, their cross-product gives us a normal direction.

$$p \times q = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 3 & 3 & 3 \\ 6 & -10 & -12 \end{vmatrix} = (-6, 54, -48) = -6(1, -9, 8),$$

so we can take $\vec{n} = (1, -9, 8)$. Then an equation for π is of the form

$$\vec{n} \cdot (x, y, z) = \vec{n} \cdot u,$$

where clearly $\vec{n} \cdot u = \vec{n} \cdot v = \vec{n} \cdot w = 7$. This gives us

$$x - 9y + 8z = 7.$$

(3) Here $p'(t) = (1, 2t, 3t^2)$, $p(2) = (2, 4, 8)$ and $p'(2) = (1, 4, 12)$. A vector equation for the tangent line is

$$L(t) = p(2) + p'(2)t, \ t \in \mathbb{R},$$

so we get

$$L(t) = (2, 4, 8) + (t, 4t, 12t), \ t \in \mathbb{R}.$$

Parametric equations are then

$$x(t) = 2 + t, \ y(t) = 4 + 4t, z(t) = 8 + 12t, \ t \in \mathbb{R}.$$

From these we can write the symmetric equations as

$$x - 2 = \frac{y - 4}{4} = \frac{z - 8}{12}.$$