



Bilkent University

Quiz # 04
Math 102 Section 09 Calculus II
4 March 2024 Monday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Evaluate the series $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{3^n}$.

Show your work in detail. Correct answers with no justification will not get any credit.

Hint: Start with the geometric series and construct an analytic function whose Taylor series looks like the above series, as we did in class!

Grading: 10 points

Solution: (Grader: melis.gezer@bilkent.edu.tr)

Start with the geometric series.

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots, \quad |x| < 1.$$

Differentiate both sides with respect to x .

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots + nx^{n-1} + \cdots, \quad |x| < 1.$$

Multiply both sides by x .

$$\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + 4x^4 + \cdots + nx^n + \cdots, \quad |x| < 1.$$

Differentiate both sides with respect to x .

$$\frac{1+x}{(1-x)^3} = 1 + 4x + 9x^2 + 16x^3 + \cdots + n^2x^{n-1} + \cdots, \quad |x| < 1.$$

Multiply both sides by x .

$$\frac{x(1+x)}{(1-x)^3} = x + 4x^2 + 9x^3 + 16x^4 + \cdots + n^2x^n + \cdots, \quad |x| < 1.$$

Then we finally have

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{3^n} = \left. \frac{x(1+x)}{(1-x)^3} \right|_{x=-\frac{1}{3}} = -\frac{3}{32}.$$