Bilkent University

Quiz # 06
Math 102 Section 09 Calculus II
18 March 2024 Monday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Let π be the plane that passes through the points $P_0 = (1, 3, 5)$, $Q_0 = (7, -6, 13)$ and is parallel to the line $\dfrac{x-1}{2} = \dfrac{y-3}{4} = \dfrac{z-5}{11}$. Write the equation of this plane in the form $Ax+By+Cz = D$ where $D \geq 0$.

Q-2) Does the line $\dfrac{x-7}{127} = \dfrac{y+6}{31} = \dfrac{z-13}{-2}$ intersect the above plane π?

If so find the intersection point. If not explain why. Does this line lie in π?

Show your work in detail. Correct answers with no justification will not get any credit.

Grading: 8+2=10 points

Solution: (Grader: `melis.gezer@bilkent.edu.tr`)

(1) The vector $\vec{u} = Q_0 - P_0 = (6, -9, 8)$ is parallel to π. On the other hand a vector parallel to the given line is, reading off from the given symmetric equations is $\vec{v} = (2, 4, 11)$. Then the vector $\vec{n} = \vec{u} \times \vec{v}$ is orthogonal to π. Finally, an equation of π will be $\vec{n} \cdot (P - P_0) = 0$, or $\vec{n} \cdot P = \vec{n} \cdot P_0$, where $P = (x, y, z)$. Now we calculate $\vec{n}$.

$$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -9 & 8 \\ 2 & 4 & 11 \end{vmatrix} = (-131, -50, 42).$$

We also check that $\vec{n} \cdot P_0 = -71$. Then an equation for π is $-131x - 50y + 42z = -71$. We multiply this by -1 to make $D \geq 0$ to obtained the required equation as

$$131x + 50y - 42z = 71.$$

(2) From the given symmetric equations of this line we see immediately that the point $Q_0 = (7, -6, 13)$ is on this line.

We can easily show that this line does not lie in π because its direction vector $\vec{w} = (127, 31, -2)$ is not orthogonal to $(A, B, C) = (131, 50, -42)$:

$$(131, 50, -42) \cdot (127, 31, -2) \neq 0.$$

In fact you need not actually find this number (which is 18271), since it is clear that it is strictly positive, which suffices to say that these vectors are not orthogonal.

Hence this line does not lie in π but intersects it at $Q_0 = (7, -6, 13)$.