Bilkent University

Quiz # 08
Math 102 Section 09 Calculus II
1 April 2024 Monday
Instructor: Ali Sinan Sertöz
Solution Key

Q-1) Consider the equation $3x^4 - 2x^2y + y^2z + xyz^3 = 65$ which defines z as a differentiable function of x and y.

(i) Find the value of $z(1,2)$.
Hint: $t^3 + 2t - 33 = (t^2 + 3t + 11)(t - 3)$.

(ii) Calculate $\left.\dfrac{\partial z}{\partial x}\right|_{(1,2)}$ and $\left.\dfrac{\partial z}{\partial y}\right|_{(1,2)}$

(iii) Write the linearization of $z(x,y)$ at the point $(x,y) = (1,2)$ in the form $L(x,y) = Ax + By + C$, where A, B and C are rational numbers.

(iv) Calculate $L(\frac{3}{2}, \frac{3}{2})$.
Note: The difference between $L(\frac{3}{2}, \frac{3}{2})$ and $z(\frac{3}{2}, \frac{3}{2})$ is $0.0032\ldots$.

Show your work in detail. Correct answers with no justification will not get any credit.

Grading: 1+4+3+2=10 points

Solution: (Grader: `melis.gezer@bilkent.edu.tr`)

(i) Putting $(x,y) = (1,2)$ into the above equation we obtain $2z^3 + 4z - 66 = 0$. From the hint we see that the only real solution to this is $z = 3$.

(ii) We apply $\dfrac{\partial}{\partial x}$ to both sides of the above equation to find $12x^3 - 4xy + y^2\dfrac{\partial z}{\partial x} + yz^3 + 3xyz^2\dfrac{\partial z}{\partial x} = 0$.
Now putting $(x,y,z) = (1,2,3)$ into this we get $\left.\dfrac{\partial z}{\partial x}\right|_{(1,2)} = -1$.

Now apply $\dfrac{\partial}{\partial y}$ to both sides of the above equation to find $-2x^2 + 2yz + y^2\dfrac{\partial z}{\partial y} + xz^3 + 3xyz^2\dfrac{\partial z}{\partial y} = 0$.
Putting $(x,y,z) = (1,2,3)$ into this we get $\left.\dfrac{\partial z}{\partial y}\right|_{(1,2)} = -\dfrac{37}{58}$.

(iii) $L(x,y) = \left(\left.\dfrac{\partial z}{\partial x}\right|_{(1,2)}\right)(x-1) + \left(\left.\dfrac{\partial z}{\partial y}\right|_{(1,2)}\right)(y-2) + z(1,2)$.

Putting in the values we found so far and simplifying we get $L(x,y) = \dfrac{153}{29} - \dfrac{37}{58}y - x$.

(iv) $L(\frac{3}{2}, \frac{3}{2}) = \dfrac{327}{116}$.
For your information, $L(\frac{3}{2}, \frac{3}{2}) = 2.818\ldots$ while $z(\frac{3}{2}, \frac{3}{2}) = 2.815\ldots$, the difference being $0.0032\ldots$.