



Quiz # 09
Math 102 Section 09 Calculus II
15 April 2024 Monday
Instructor: Ali Sinan Sertöz
Solution Key

Bilkent University

Q-1) Let $z = z(x, y)$, $x = x(u, v)$, $y = y(u, v)$, $u = u(s, t)$ and $v = v(s, t)$ be differentiable functions of their variables. We have the following data:

$u(3, 7) = 9$	$v(3, 7) = 11$	$u(7, 3) = -9$	$u(7, 3) = -10$
$u_s(3, 7) = 12$	$v_s(3, 7) = -1$	$u_t(3, 7) = 2$	$v_t(3, 7) = -3$
$x(9, 11) = -3$	$y(9, 11) = 3$	$x(3, 7) = 12$	$y(3, 7) = -8$
$x_u(9, 11) = 4$	$y_u(9, 11) = 5$	$x_u(3, 7) = -12$	$y_u(3, 7) = 6$
$x_v(9, 11) = -6$	$y_v(9, 11) = 7$	$x_v(3, 7) = 4$	$y_v(3, 7) = 14$
$z(-3, 3) = 16$	$z(9, 11) = 17$	$z(3, 7) = 18$	$z(12, -8) = 19$
$z_x(-3, 3) = -2$	$z_y(-3, 3) = 9$	$z_x(9, 11) = 2$	$z_y(9, 11) = -9$
$z_x(3, 7) = 5$	$z_y(3, 7) = 13$	$z_x(12, -8) = -5$	$z_y(12, -8) = -7$

Fill in the following boxes with numbers using this table. No questions asked!

$$\left. \frac{\partial x}{\partial s} \right|_{(s,t)=(3,7)} = \boxed{54}$$

$$\left. \frac{\partial y}{\partial t} \right|_{(s,t)=(3,7)} = \boxed{-11}$$

$$\left. \frac{\partial z}{\partial s} \right|_{(s,t)=(3,7)} = \boxed{369}$$

$$\left. \frac{\partial z}{\partial t} \right|_{(s,t)=(3,7)} = \boxed{-151}$$

The linearization of z as a function of x and y at the point $(x, y) = (12, -8)$ is

$$L(x, y) = \boxed{23} + \boxed{-5}x + \boxed{-7}y$$

The linearization of z as a function of s and t at the point $(s, t) = (3, 7)$ is $(s, t) = (3, 7)$ is

$$L(s, t) = \boxed{-34} + \boxed{369}s + \boxed{-151}t$$

Grading: Each correctly filled box is 1 point. Grader: melis.gezer@bilkent.edu.tr

Solution:

$$\left. \frac{\partial x}{\partial s} \right|_{(s,t)=(3,7)} = \left. \frac{\partial x}{\partial u} \right|_{(u,v)=(9,11)} \left. \frac{\partial u}{\partial s} \right|_{(s,t)=(3,7)} + \left. \frac{\partial x}{\partial v} \right|_{(u,v)=(9,11)} \left. \frac{\partial v}{\partial s} \right|_{(s,t)=(3,7)} = (4)(12) + (-6)(-1) = 54$$

$$\left. \frac{\partial x}{\partial t} \right|_{(s,t)=(3,7)} = \left. \frac{\partial x}{\partial u} \right|_{(u,v)=(9,11)} \left. \frac{\partial u}{\partial t} \right|_{(s,t)=(3,7)} + \left. \frac{\partial x}{\partial v} \right|_{(u,v)=(9,11)} \left. \frac{\partial v}{\partial t} \right|_{(s,t)=(3,7)} = (4)(2) + (-6)(-3) = 26$$

$$\left. \frac{\partial y}{\partial s} \right|_{(s,t)=(3,7)} = \left. \frac{\partial y}{\partial u} \right|_{(u,v)=(9,11)} \left. \frac{\partial u}{\partial s} \right|_{(s,t)=(3,7)} + \left. \frac{\partial y}{\partial v} \right|_{(u,v)=(9,11)} \left. \frac{\partial v}{\partial s} \right|_{(s,t)=(3,7)} = (5)(12) + (7)(-1) = 53$$

$$\left. \frac{\partial y}{\partial t} \right|_{(s,t)=(3,7)} = \left. \frac{\partial y}{\partial u} \right|_{(u,v)=(9,11)} \left. \frac{\partial u}{\partial t} \right|_{(s,t)=(3,7)} + \left. \frac{\partial y}{\partial v} \right|_{(u,v)=(9,11)} \left. \frac{\partial v}{\partial t} \right|_{(s,t)=(3,7)} = (5)(2) + (7)(-3) = -11$$

$$\left. \frac{\partial z}{\partial s} \right|_{(s,t)=(3,7)} = \left. \frac{\partial z}{\partial x} \right|_{(x,y)=(-3,3)} \left. \frac{\partial x}{\partial s} \right|_{(s,t)=(3,7)} + \left. \frac{\partial z}{\partial y} \right|_{(x,y)=(-3,3)} \left. \frac{\partial y}{\partial s} \right|_{(s,t)=(3,7)} = (-2)(54) + (9)(53) = 369$$

$$\left. \frac{\partial z}{\partial t} \right|_{(s,t)=(3,7)} = \left. \frac{\partial z}{\partial x} \right|_{(x,y)=(-3,3)} \left. \frac{\partial x}{\partial t} \right|_{(s,t)=(3,7)} + \left. \frac{\partial z}{\partial y} \right|_{(x,y)=(-3,3)} \left. \frac{\partial y}{\partial t} \right|_{(s,t)=(3,7)} = (-2)(26) + (9)(-11) = -151$$

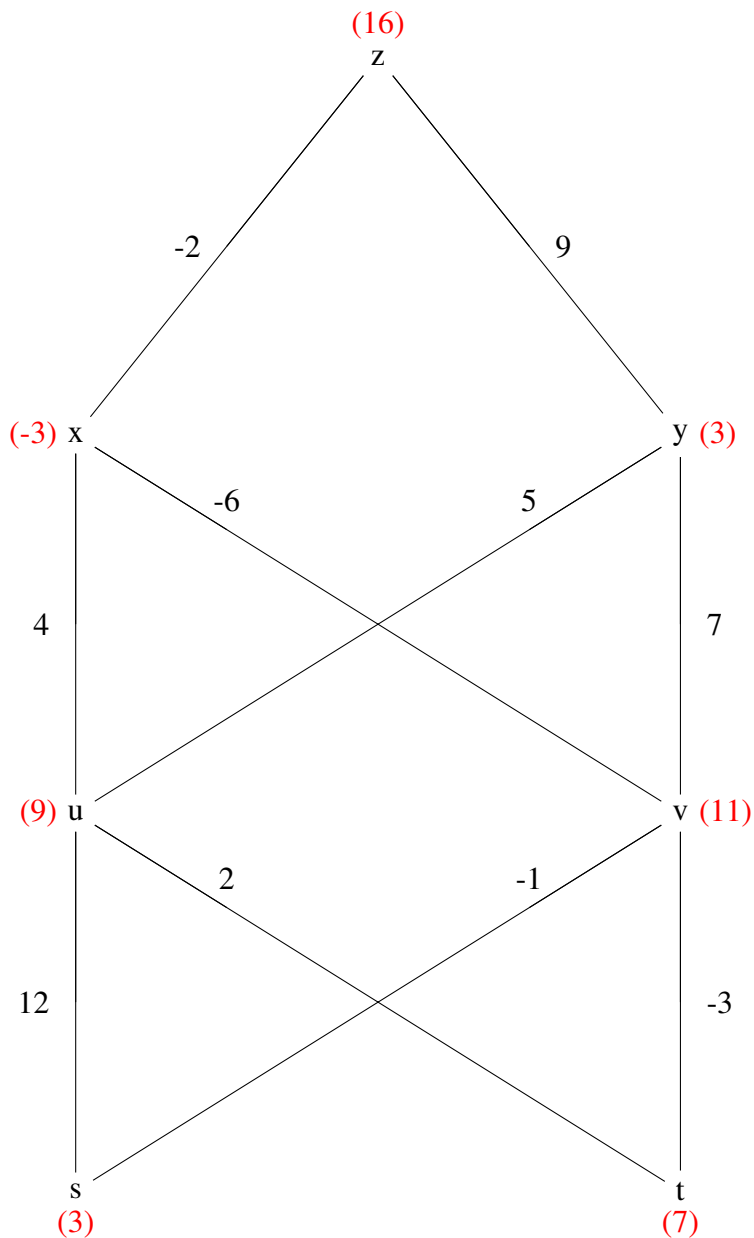
The linearization of z as a function of x and y at the point $(x, y) = (12, -8)$ is

$$\begin{aligned} L(x, y) &= \left(z \Big|_{(x,y)=(12,-8)} \right) + \left(\frac{\partial z}{\partial x} \Big|_{(x,y)=(12,-8)} \right) (x - 12) + \left(\frac{\partial z}{\partial y} \Big|_{(x,y)=(12,-8)} \right) (y + 8) \\ &= (19) + (-5)(x - 12) + (-7)(y + 8) \\ &= 23 - 5x - 7y. \end{aligned}$$

The linearization of z as a function of s and t at the point $(s, t) = (3, 7)$ is

$$\begin{aligned} L(s, t) &= \left(z \Big|_{(s,t)=(3,7)} \right) + \left(\frac{\partial z}{\partial s} \Big|_{(s,t)=(3,7)} \right) (s - 3) + \left(\frac{\partial z}{\partial t} \Big|_{(s,t)=(3,7)} \right) (t - 7) \\ &= (16) + (369) (s - 3) + (-139) (t - 7) \\ &= -34 + 369s - 151t. \end{aligned}$$

An alternate approach is on next page.



The number in parenthesis near a variable is its value at the relevant point. The number adjacent to a line is the value of the relevant partial derivative at that point.

Now for example to calculate $\frac{\partial z}{\partial s} \Big|_{(s,t)=(3,7)}$ follow all the paths from z to s and for each such path multiply the numbers you see adjacent to the lines you follow. Then add all these numbers.

$$\frac{\partial z}{\partial s} \Big|_{(s,t)=(3,7)} = (-2)(4)(12) + (-2)(-6)(-1) + (9)(5)(12) + (9)(7)(-1) = -96 - 12 + 540 - 63 = 369.$$