Math 113 Homework 7 • Due: 31 December 2004 Friday class hour. • Instructor: Sertöz

Q-1) Let $T(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$ be the Taylor polynomial of degree 4 of $\tan x$. Find the coefficients $a_0, \ldots, a_5$.

Solution: $T(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5.$

Q-2) Let $S(x) = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4$ be the Taylor polynomial of degree 4 of $\frac{x}{T(x)}$ where $T(x)$ is as in question 1 above. Find the coefficients $b_0, \ldots, b_4$.

Solution: $S(x) = 1 - \frac{1}{3}x^2 - \frac{1}{45}x^4.$

Q-3) Let $R(x) = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4$ be the Taylor polynomial of degree 4 of $x \cot x$. Find the coefficients $c_0, \ldots, c_4$.

Solution: $R(x) = 1 - \frac{1}{3}x^2 - \frac{1}{45}x^4.$

Q-4) Compare the polynomials $S(x)$ and $R(x)$. Are they the same? Explain why. Are they different? Explain why.

Solution: If $F(x) = \frac{1}{H(x)}$, then $F^{(n)}(0)$ depends on $H(0), H'(0), \ldots, H^{(n)}(0)$. The first n derivatives of the Taylor polynomial $T(x)$ of $H(x)$ agree with those of $H(x)$. Therefore the first n derivatives of $\frac{1}{T(x)}$ are the same as those of $F(x)$.

In our case first check that $T(x)/x$ is the Taylor polynomial of $\tan x/x$. Then observe that $S(x)$ is the Taylor polynomial of $1/(\tan x/x)$ and $R(x)$ is the Taylor polynomial of $1/(T(x)/x)$, so they are the same.
