Math 113 – Homework 7 – Solutions

Due: 20 December 2005 Tuesday.

Q-1) $$\lim_{x\to 0}\frac{15\tan x-15x-5x^3-2x^5}{24x\cos x-24x+12x^3-x^5}\overset{Taylor}{=}\lim_{x\to 0}\frac{15(x+\frac{x^3}{3}+\frac{2x^5}{15}+\frac{17x^7}{315}+\cdots)-15x-5x^3-2x^5}{24x(1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\cdots)-24x+12x^3-x^5}$$

$$=\lim_{x\to 0}\frac{\frac{17x^7}{21}+\cdots}{-\frac{x^7}{30}+\cdots}=-\frac{170}{7}.$$

Q-2) $$\lim_{x\to 0}\frac{2x^3\sec x}{2x-x^2-2\ln(1+x)}\overset{Taylor}{=}\lim_{x\to 0}\frac{2x^3\sec x}{2x-x^2-2(x-x^2/2+x^3/3-\cdots)}$$

$$=\lim_{x\to 0}\frac{2x^3\sec x}{-2x^3/3+\cdots}=-3.\quad \text{(Note here that } \sec 0=1.)$$

Q-3) $$\lim_{x\to 0}\frac{(e^{\sin x}-1)\sin x^3}{x^3\tanh x}=\lim_{x\to 0}\frac{(e^{\sin x}-1)}{\tanh x}\lim_{x\to 0}\frac{\sin x^3}{x^3}\overset{L'H\hat{o}pital}{=}\lim_{x\to 0}\frac{e^{\sin x}\ \cos x}{\operatorname{sech}^2 x}\cdot 1=1.$$

Q-4) $$\lim_{x\to 0}\frac{\arctan x}{\sinh x\cos x}=\lim_{x\to 0}\frac{\arctan x}{\sinh x}\lim_{x\to 0}\frac{1}{\cos x}\overset{L'H\hat{o}pital}{=}\lim_{x\to 0}\frac{\frac{1}{1+x^2}}{\cosh x}\cdot 1=1.$$

Q-5) Find $\displaystyle\lim_{x\to 0}\frac{e^{-1/x^2}}{x^n}$, where n is any integer.

If $n\le 0$, then this limit is clearly 0. So we now assume that $n>0$.

$$\lim_{x\to 0}\frac{e^{-1/x^2}}{x^n}\overset{(t=\frac{1}{x})}{=}\lim_{t\to\pm\infty}\frac{t^n}{e^{t^2}}\overset{L'H\hat{o}pital}{=}\lim_{t\to\pm\infty}\frac{nt^{n-1}}{2te^{t^2}}$$

$$=\lim_{t\to\pm\infty}\frac{n}{2t}\lim_{t\to\pm\infty}\frac{t^{n-1}}{e^{t^2}}=\cdots=\lim_{t\to\pm\infty}\frac{n!}{2^nt^n}\lim_{t\to\pm\infty}\frac{1}{e^{t^2}}=0.$$

Conclusion: $\displaystyle\lim_{x\to 0}\frac{e^{-1/x^2}}{x^n}=0$, where n is any integer.

`Comments and questions to sertoz@bilkent.edu.tr`