Calculus 113 Homework 4

Due date: 2 November 2007 Friday

Please take your homework solutions to room SA144, Ali Adalı's office.

Q-1) Find the largest $\delta > 0$ satisfying the property that for all $x, y \in [0, 100]$ with $|x - y| < \delta$, we have $|x^2 - y^2| < 1$. Show that no such $\delta > 0$ exists if we choose $x, y \in [0, \infty)$.

Q-2-a) Find a function f which is continuous and bounded but not uniformly continuous on $(0, 1]$.

Q-2-b) Find a function f which is continuous and bounded but not uniformly continuous on $[0, \infty)$.

Q-3-a) For any non-negative integer $n \in \mathbb{N}$, find a function $f : \mathbb{R} \to \mathbb{R}$ such that $f^{(n)}(x)$ exists and is continuous for all $x \in \mathbb{R}$, but $f^{(n+1)}(0)$ does not exist.

Q-3-b) For any non-negative integer $n \in \mathbb{N}$, find a function $f : \mathbb{R} \to \mathbb{R}$ such that $f^{(n)}(x)$ exists for all $x \in \mathbb{R}$, but $f^{(n)}(x)$ is not continuous at $x = 0$.

Q-4-a) Consider the function

$$f(x) = \begin{cases} x^2 \sin^2(1/x) & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Show that $x = 0$ is a local minimum for f but f is neither decreasing to the left of 0 nor increasing to the right of it.

Q-4-b) Consider the function

$$f(x) = \begin{cases} \alpha x + x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

where $0 < \alpha < 1$. Show that $f'(0) = \alpha > 0$ but f is not increasing on any open interval containing 0.