Due on April 3, 2006, Monday, Class time. No late submissions!

MATH 114 Homework 6

1: Let $f(x, y) = 8x^3 + y^3 + 6xy$. Find local min/max, global min/max and saddle points, if they exist, for this function.

2: Let $f(x, y) = xy + 2x - \ln(x^2 y)$ where $x, y > 0$. Find local min/max, global min/max and saddle points, if they exist, for this function.

3: Let $f(x, y) = x^2 + kxy + y^2$ where $k \in \mathbb{R}$. Find local min/max, global min/max and saddle points, if they exist, for this function, for each value of k.

4: Find the distance from the surface $z = x^2 + y^2 + 10$ to the plane $x + 2y - z = 0$. (This means you will calculate the minimum distance $|p - q|$ where p is on the surface and q is on the plane.)

5: Consider the surface S given by $f(x, y, z) = 0$ and assume that $p_0 = (x_0, y_0, z_0)$ is on the surface with $\frac{\partial f}{\partial z}(p_0) \neq 0$.

(i) Write the equation of the tangent plane to the surface S at p_0. From the equation of the tangent plane solve for z. Geometrically this is the linear approximation for the surface at the point p_0.

(ii) Now consider z as a function of the two independent variables x and y, say $z = g(x, y)$ with $z_0 = g(x_0, y_0)$. Assume as above that $f(x, y, g(x, y)) = 0$. Write a linear approximation for g at (x_0, y_0). i.e. write

$$L(x, y) = g(x_0, y_0) + \frac{\partial g}{\partial x}(x_0, y_0)\ (x - x_0) + \frac{\partial g}{\partial y}(x_0, y_0)\ (y - y_0).$$

Algebraically this is the linear approximation of the surface at the point p_0. How does this compare to the one found in the previous part? (This means you must calculate the partial derivatives of g in terms of the partial derivatives of f at the point p_0.)